HEC Montréal

APPLICATION OF GAME THEORY TO GLOBAL ENVIRONMENTAL PROBLEMS

by

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A thesis submitted in partial fulfillment of the requirements of the degree of
Philosophiae Doctor (Ph.D.)
in Applied Economics, at HEC Montréal

Montreal, Quebec, Canada
October 2004

Université de Montréal
Faculté des études supérieures

This thesis entitled:

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RÉSUMÉ

Cette thèse consiste en trois essais traitant de problèmes environnementaux à l'échelle internationale. La méthodologie utilisée consiste principalement en l'application de la théorie des jeux coopératifs, des jeux dynamiques et du contrôle optimal.

Le premier essai traite de l'un des principaux problèmes environnementaux à l'échelle mondiale, à savoir la destruction des forêts tropicales. Pour analyser et résoudre ce probleme nous considérons deux joueurs ayant des utilités différentes pour la conservation des forêts qui est considéré comme une source de revenu pour l'un et un moyen de protéger l'environmment pour l'autre. Le premier joueur (le Nord) représente un ensemble de pays développés ayant pour objectif la maximisation de la taille de la forêt à la fin d'une horizon temporelle fixée. Le deuxième joueur (le *Sud*) essai de maximiser ses revenus traduisant un arbitrage entre l'exploitation de la forêt et les revenus tirés des activités agricoles. Notre objectif consiste à étudier la possibilité d'une coopération entre les pays du *Sud* et les pays du *Nord*, où les pays du *Nord* offrent un transfert pour compenser la perte de revenus que les pays forestiers (du *Sud*) peuvent encourir suite à une réduction du taux de déforestation. Pour ce faire nous avons étudié deux scénarios. Dans le premier, on suppose une politique de *laisser-faire*, où le *Sud* résout un problème de contrôle optimal sur horizon fini sans aucune intervention (ou transfert) de la part du *Nord*. Le revenu et l'exploitation optimale de forêt obtenus représentent un repère pour le second scénario, dans lequel le *Nord* offre des subventions au *Sud* pour l'encourager à réduire son taux de déforestation. Les deux scénarios sont alors analysés et comparés en termes de stratégies, de résultats et de conservation de la forêt. Les résultats montrent que l'amélioration du bien-être du *Sud* n'est pas toujours assurée avec l'introduction de la possibilité de transferts. De même, le stock de forêt n'est pas toujours plus grand dans le cas où le *Nord* fait face à une contrainte budgétaire serrée. Par conséquent l'applicabilité de ce mécanisme

de transfert n'est pas toujours garantie comme le prédisent les études antérieures.

Le deuxième essai porte lui aussi sur le problème de déforestation dans les pays en voie de développement. Il vise à élaborer des stratégies incitatives qui pourraient être employées par les pays du *Nord* afin d'amener les pays forestiers (pays du *Sud*) à choisir une politique de déforestation qui soit soutenable dans le long terme (durable) même s'ils maximisent leur profit sur un horizon fini. Le mécanisme d'incitation consiste à offrir des transferts qui dépendent directement de la stratégie de déforestation déployée dans les pays du *Sud*. Ces stratégies sont calculées de manière intrinsèque pour que la solution optimale au problème de maximisation du bien-être du *Sud* coïncide avec la politique de déforestation désirée.

Finalement le troisième article traite d'un autre aspect des problèmes environnementaux se posant à l'échelle internationale, à savoir problème de Free-riding (ou resquillage) et la stabilité des coalitions dans le contexte de l'élaboration d'un accord environnemental international visant le contrôle de la pollution (à travers la réduction des émissions des gaz à effet de serre). Notre objectif principal consiste à étudier la question de réconciliation entre deux approches différentes qui ont étudié cette question.

Dans la littérature existante, la pratique la plus courante consiste à considérer le nombre de pays participants dans l'accord comme une donnée exogène : elle part du principe que tous les pays forment une grande coalition et calcule le gain total émanant de cette coopération. Ce dernier représente la différence entre le coût total de la réduction des émissions des gaz à effet de serre en cas de coopération globale et en cas de non-coopération entre les différents pays. Ce gain est ensuite réparti entre tous les signataires de l'accord, et la difficulté principale consiste alors à trouver la méthode de partage qui garantisse la stabilité de cet accord au sens du 'Noyau'. Ainsi, la solution de partage doit garantir à chaque joueur une amélioration par rapport au cas où il jouerait seul ou ferait partie d'une sous-coalition de taille inférieure à la grande coalition initiale, et ce dans le but d'éviter qu'un ou plusieurs pays dévient de l'accord.

Dans d'autres travaux, par contre, les chercheurs ont considéré le nombre de pays comme une variable endogène, tenant ainsi compte de la possibilité de formation de petites coalitions. Pour être "stables", ces coalitions doivent alors satisfaire des conditions de stabilité interne et de stabilité externe telles que définies originellement par d'Aspremont et Gabzewicz (1986) pour étudier les problèmes de cartel en organisation industrielle. Par la stabilité interne, on entend qu'aucun pays de la coalition formée n'a intérêt à la quitter; c'est le test de sortie. La stabilité externe quand à elle signifie qu'aucun des pays qui sont à l'extérieur de cette coalition n'a intérêt à la rejoindre; c'est le test d'entrée. Selon cette approche, le nombre de pays qui peuvent former une coalition stable, et par conséquent signer un accord international, est très limité, même en introduisant des transferts monétaires latéraux.

La comparaison entre ces deux approches a été faite par Tulkens (1998), qui a laissé transparaître un espoir de réconciliation en utilisant la fonction caractéristique.

Notre but dans cet essai est d'étudier les propriétés devant être satisfaites par une telle fonction caractéristique et de vérifier son existence en utilisant la méthodologie des jeux coopératifs. Nous avons donc étudié le problème en utilisant les différents point de vue concernant le concept de stabilité, en analysant trois définitions possibles de la fonction caractéristique, à savoir l'équilibre de Nash, l'équilibre de Nash partiel et la solution de von Newman Morgenstern. La confrontation des résultats que nous avons obtenus dans les deux scénarios indique qu'il est impossible de contourner le problème de resquillage pour avoir une grande coalition stable, même si les joueurs adoptent des stratégies non-coopératives. Réconcilier les deux approches reste alors impossible, contrairement à la conjecture de Tulkens (1998).

Mots clés: accords internationaux sur l'environnement,déforestation tropicale, exploitation durable, fonction caractéristique, jeux differential, jeux cooperatif, transferts monétaires, mécanisme incitatif, free-riding.

SUMMARY

This dissertation consists of three essays that deal with important global environmental problems, using a game theoretical framework.

The first essay deals with tropical deforestation as a global environmental issue and studies the possibility of an agreement between developing countries (or the *South*) and developed countries (or the *North*) to reduce the deforestation rate using a subsidy program. For this purpose, we study two scenarios: the first one is a *laisser-faire* policy, where the *South* solves an optimal control problem over a finite horizon; and the second is a Stackelberg game, where the *North* offers subsidies to the *South* in order to reduce the deforestation rate. The two scenarios are then analyzed and compared in terms of strategies, outcomes and forest conservation. In contrast to previous studies, our final results show that the subsidy program cannot be unconditionally implemented, in the sense that some conditions have to be satisfied to guarantee the *South*'s participation and in some cases the *North*'s implication in this program.

The second essay is also concerned with tropical deforestation in developing countries. The objective of this essay is to determine incentive strategies for the *South*, conditioning the *North*'s transfers directly on the *South*'s actions regarding forest exploitation. These strategies can be used by the *North* to indirectly force the *South* to choose an optimal deforestation policy which is sustainable in the long run.

Finally, in the last essay, we investigate the possibility of reconciling two different approaches regarding the design of an International Environmental Agreement. We first study the problem from the cooperative games perspective, on which the first approach is based, by analyzing three definitions of the characteristic function. Then, we address the issue of free-riding and stability of coalitions as defined by the second approach. Our results show that it is not feasible to reconcile the two approaches, which differs from Tulkens's 1998 conjecture.

Key words: Characteristic Function, Free-riding, Transfers, Incentive Mechanism, International Environmental Agreements, differential games, cooperfative game theory, Tropical Deforestation, Sustainability.

Contents

"Only when the last tree has died and the last river been poisoned and the last fish been caught will we realize we cannot eat money." ˜Cree Indian Proverb

À mes parents avec amour et reconnaissance

ACKNOWLEDGMENTS

I would like to thank my advisers Dr. Michèle Breton and Dr. Georges Zaccour for their support and helpful comments that contributed to guide me through the dissertation process.

I am also grateful to all other persons that contribute to improve the content of this thesis. Especially, I thank Dr. Guiomar Martìn-Herràn for her generous assistance and continuous help.

I also want to express my thankfulness to all the committee members for accepting to read and comment this work. I, especially, thank Dr. Peter Kort for his pertinent comments, and Dr. Désiré Vencatachellum for his constructive comments and his guidance during the course of my studies.

I thank my friends whose support and friendship have given me courage and perseverance to get through some of the difficult moments.

A special thanks to Michel for his unrelenting support, and for being my inspiration in moments of doubt.

Finally, I want to express my deepest gratitude and love to my family for the unconditional and constant support.

REMERCIEMENTS

Je remercie mes directeurs de recherche, professeur Georges Zaccour et professeur Michèle Breton, pour leurs encouragements et leurs directives qui m'ont guidé le long de mon parcours de thèse.

Je suis également reconnaissante envers toutes les personnes qui, de quelque manière que ce soit, ont contribué à améliorer le contenu de cette thèse.

En particulier, je remercie professeure Guiomar Martìn-Herràn pour sa généreuse assistance et continuelle aide.

Mes remerciements se dirigent également à tous les membres du jury pour avoir accepté de lire et commenter ce travail. Plus particulièrement, je tiens à remercier professeur Peter Kort pour ces pertinants commentaires, et professeur Désiré Vencatachellum pour ses commentaires constructifs et ses précieux conseils tout le long de mon parcours d'études doctorales.

Je remercie mes amis qui à travers leurs encouragements, conseil et amitié m'ont donné le courage et le sens de persévérance nécessaires pour traverser les moments difficile de ce parcours.

Je remercie d'une manière toute spéciale Michel pour son précieux support et pour m'avoir inspiré pendant les moments de doute.

Finalement, je tiens à exprimer ma profonde gratitude envers ma famille pour son appui continu et inconditionnel.

INTRODUCTION

This dissertation consists of three essays that deal with important global environmental problems, using a game theoretic framework. All three essays construct game theoretic models. Depending on the context of the problem we refer to cooperative or non-cooperative game theory and also consider static or dynamic games, in particular differential games.

The first and second chapters address the global environmental issue of deforestation in developing countries as a *North-South* game, using differential games and optimal control theory as methodological framework. The last essay focuses on coalition formation and stability of International Environmental Agreements (IEA), studied from cooperative and non-cooperative game theory perspectives in a static framework.

Forest destruction in Southern countries is alarming in light of its adverse consequences for biodiversity and climate change. However, for the forestry countries, forest exploitation represents a necessary source of revenue (timber production and agricultural use of the converted land). Hence, an external action is needed from developed countries, or the international community, to reduce forest destruction. Indeed, besides the reduction of greenhouse gases emission, forest preservation and regeneration presents one of the major instruments proposed to counter global warming through carbon dioxide sequestration. An eventual cooperation between the *North* and the *South* is required and was encouraged by the United Nations Framework Convention on Climate Change (UNFCCC) and the Kyoto Protocol. The *North-South* interaction to reduce deforestation is advocated in the literature through two main policies: The first is trade measures to encourage sustainable timber production; and the second is transfer payment used as a revenue compensation from the *North* to the *South* to encourage them to reduce forest exploitation. In this thesis, we focus on the second solution.

The first essay presents a subsidy program that can be employed by the *North* to reduce the deforestation pace in developing countries. The motivation of this essay is to determine the conditions under which a potential agreement can be signed between the *North* and the *South* to reduce forest destruction.

The methodology used in this essay is based on game theoretic models and uses optimal control techniques. The problem is addressed as a Stackelberg game, where the developed countries (or the *North*) act as a leader and propose a subsidy rate, and developing countries (or the *South*) play as a follower choosing the optimal deforestation rate.[1]Regarding the information structure, we retain the tractable open-loop strategies rather than its conceptually appealing closed-loop counterpart. This choice is justified because we consider a short horizon for the aid program. Furthermore, knowing that the *South* will work out its deforestation policy according to the subsidy rate announced by the North, the latter would not jeopardize its future credibility by retracting at an intermediate date from what it announced at the initial instant of the game. On the other hand, in order to examine the possibility of implementation of the coordinated effort solution, we compare it with a benchmark scenario: a *laisser-faire* solution, where the *South* maximizes its welfare in absence of the North's intervention, computed using an optimal control formulation.

In this essay we consider that the total amount of transfers that can flow from the Northern community to the Southern countries is limited, as the North faces a budget

[1]Throughout this text the developed countries will be known as the North and the developing countries as the South.

constraint, which is new in the field as in the previous related literature. We also differ from previous literature by using a finite horizon setting rather than an infinite horizon one. Our aim is to design a subsidy program that can be implemented and profitable in the short term, given the assumed irreversibility of deforestation and the immediacy of the problem. Contrary to the previous literature, our results show that transfers do not necessarily improve the welfare of the forestry countries. We also show that the forest cannot be always improved when the North is subject to a binding budget constraint.

The second essay presents incentive mechanism settings to enforce sustainable forest management in the South. The objective of this essay is to determine incentive strategies conditioning the funds' transfers directly by the South's actions regarding forest exploitation. These strategies can be used by the North to indirectly compel the South to choose an optimal deforestation policy which is sustainable in the long run.

For this purpose, we define a transfer function that guarantees the participation of the recipient country by compensating it for the total loss due to a better forest conservation. Indeed, this transfer designs an incentive mechanism ensuring that the forestry country will respect its engagement regarding a sustainable forest exploitation. To our knowledge this kind of incentive mechanism has never been used previously in sustainable forest management literature. To our knowledge, to date, this kind of incentive mechanism has never been used in the sustainable forest management literature. It is, however, a well known instrument in game theory, of which the application to environmental games and resource management is far from new.

Finally, the last essay deals with the stability of coalitions in games of pollution control. It studies the design of an international environmental agreement, which is a mechanism allocating to each country a suitable emissions policy supported possibly by a monetary transfer. This has been addressed in the literature following two lines of thought, that is cooperative or non-cooperative game theory approaches ending up with different results regarding the size of a possible stable coalition. The challenge is to find a possible reconciliation of the two approaches. In other words, we are looking for a payment function which ensures the formation and stability of a large coalition, even if we consider that different countries are playing non-cooperatively and are acting only for their own interest.

This discussion captures what has been observed regarding the evolution of the Kyoto Protocol since it has been signed by many of the world's nations in Japan in 1997, later on rejected by United States in 2001, and more recently deferred by Russia. Our results show indeed that it is difficult to deter free riding and that no large coalition can emerge if countries decide to play non-cooperatively.

ESSAY 1

Slowing Deforestation Pace through Subsidies : A Differential Game

1. Introduction

The adverse consequences of tropical deforestation in the South on biodiversity conservation and climate change have made rainforest destruction one of the main environmental issues of international concern. Scientists have deployed considerable efforts during the last two decades in modeling the problem and assessing its importance, its causes and consequences with the aim of advancing some solutions that national and transnational agencies could implement to slow deforestation.

This essay is concerned with the design of an aid program by the North to help the South in keeping the tropical forests. The methodological framework is of two-player differential games with the rich countries acting as leader in a Stackelberg game.

The major cause of tropical deforestation seems to be the conversion of forested land to agricultural use (e.g., Amelung and Diehl, 1992; Barbier et al., 1991; Barbier and Burgess, 1997; Kaimowitz and Angelsen, 1999; Southgate et al., 1991; and Southgate, 1990). It is easily understood that since a country gets revenue from agriculture, the temptation is high to follow a *laisser-faire* policy when it comes to deforestation.

Scholars are pointing out that forest conservation is not only a cost but also has its domestic benefits such as, e.g., the avoidance of the erosion-generating changes (see, for example, Montgomery, 2002; Chomitz and Kumari, 1998; Cline, 1992). Furthermore, they argue that the costs of forest preservation are small compared to the large non-economic benefits from doing so.

At a domestic level, these benefits may, however, be much smaller compared to the global ones, implying the need of global sources to finance the forest conservation. Barret (1994-a), Von Amsberg (1994), Van Soest (1998) and Van Soest and Lensink

(2000) argue that in some instances the allocation of forest lands to alternative use may enhance the domestic country's welfare, while decreasing welfare from the global perspective. This confers to the deforestation issue its international externality dimension and makes forest conservation a global environmental issue. As a matter of fact, one hundred nations have agreed on a treaty involving both The North and the South to link environmental and economic issues in 1992 at the Earth Summit in Rio de Janeiro. This treaty triggered a stream of research which aims at coordinating the efforts of the North and the South to tackle global environmental issues such as deforestation. The mechanisms proposed can be divided into *trade measures* and *financial transfers*.

In the first perspective, the general idea is to use trade policy as an instrument to combat deforestation by affecting the level or the method of forestry activities as proposed, for example, by Barbier and Rauscher (1994), Jepma (1995) and Kolk (1996). The level of deforestation is affected by acting on the demand through the imposition of tropical timber import bans and tariffs. The methods of deforestation are controlled by discriminating between sustainable and unsustainable timber production. They encourage the first kind of production through subsidization and discourage the second by imposing selective bans and tariffs.

However, these trade interventions have been criticized because they can lead to more overextraction and a definition of property rights is proposed as an alternative to regulation. This solution, which goes up to Coase (1960) which suggests that well defined property rights could overcome the problems of externalities (see Coase Theorem), has been applied to this specific issue in recent work; for example, by Chichilnisky (1994) in an optimal control theory approach, and by Tornell and Velasco

(1992) in a differential game approach . Other authors such as Cabo et al. (2001, 2002), proposed capital transfer as a trade measure to incite biodiversity preservation using a differential game approach.

In this essay, we are mainly interested in the second perspective, which has considered aid donation and transfers as a solution for this global issue. Barbier and Rauscher (1994), using optimal control theory, compare the trade and transfer perspectives and show that financial transfers can be more effective than trade intervention to combat deforestation. These authors consider lump-sum aid donations to allow forest conservation indirectly, by reducing the necessity to exploit the forests. The results indicate that direct international transfers will unambiguously increase forest stock in the long run equilibrium.

However, lump-sum transfers have been criticized as being a passive instrument to combat deforestation. A more active way to use transfers would indeed be to make the amount of transfer conditional to the recipient country's effort to improve forest conservation. Stähler (1996) shows that paying a fixed price per unit of forest land conserved unambiguously improves the long run forest conservation. However, he adds that this kind of dependency between the transfers and the forest stock is unreliable since the smaller the forest land becomes, the more the international community is willing to pay in order to prevent the conversion of an additional unit of forest land. Thus, suspecting this mechanism, the forest countries can use their "market power" to influence the per-unit compensation through their deforestation behavior. Consequently, per-unit compensation may have adverse effects in the sense that, if a strategic behavior is employed by the recipient countries, long run forest size can turn lower than if no compensation is given. Therefore, this author raises the point that the specification of the compensation function is very important.

Mohr (1996) emphasizes this point using a bargaining game. He argues that the credibility problem about whether the donor community is indeed "hard nosed" about the specified compensation per unit of land, can give an incentive for the recipient country to increase its deforestation rate. He lays stress on the fact that the rainforest ecosystem is, to a large extent, irreversible, which makes this consequent increase in the rate of deforestation undesirable. For this reason, it may be advantageous to let the transfer depend on the deforestation rate, so that the recipient countries are directly confronted to the results of their land use decisions, as it was proposed by Van Soest and Lensink (2000). Using a contract approach, the latter derive an aid donation function that best serves the donor community's interest in terms of forest conservation. That is, the donor community offers an aid contract that establishes the terms of conditionality, after which the recipient country chooses the rate of deforestation to maximize its utility given the terms of the contract. The results indicate that both donor and recipient countries are better off under the contract aid scheme. Also, long-run forest conservation can be improved by linearly punishing deforestation or/and rewarding forest conservation using a fixed per-unit compensation price. However, to improve short-run forest conservation in a direct way, the donor should punish deforestation at an increasing rate.

Martín-Herrán et al. (2002) reconsidered the same problem in a differential game approach. This differs from the contract approach mainly because both the South (recipient countries) and the North (donor countries) decide on their respective control variable, i.e. transfers and the deforestation rate, without a direct restriction on the other player's decision variable in their optimization problems. The numerical simulations show that the welfare and/or the forest conservation can be improved under the game approach compared to the contract one.

From the above account of related literature, and to point out the differences with the closest papers to this one, i.e. Van Soest and Lensink (2000) and Martín-Herrán et al. (2002), we make the following comments.

- The fact that the recipient country could react strategically to the aid program of the donor community, points toward using game theory as an analytical framework to analyze the problem of deforestation. Such a problem is inherently dynamic and hence the adoption of differential games is natural. This contrasts the optimal control formulation in Van Soest and Lensink (2000).

- In our study the financial transfers are bounded by a budget constraint faced by the North. This makes our approach very different from the ones mentioned above. The transfer function itself seems to influence greatly the results in terms of welfare and forest conservation. We shall consider one which takes into account both the deforestation rate (the control of the South) and the forest size (the state variable).

- We consider a finite horizon model rather than an infinite horizon. Indeed, although deforestation is a long term environmental problem, the assumed irreversibility of conversion of forest to other uses invites the design of a program which is meant to have an impact in the short run. For this reason, we shall consider the implementation of an aid program available for, at most, a given period of time and whose objective, from donor perspective, is easily measurable, that is to maximize the size of forest at the final date of the program. Martín-Herrán et al. (2002) consider infinite-horizon games and utility functions for the North which are conceptually appealing but rather difficult to assess in practice. The same difference is noticeable with Van Soest and Lensink (2000).

The results indicate that making the transfer function dependent on the deforestation rate directly in addition to be dependent on the forest stock has a clear impact on (slowing) deforestation. This result is consistent with the findings in Van Soest (1998) and Van Soest and Lensink (2000). However, contrary to the previous studies, we show that when the budget constraint is binding, using a subsidy program does not necessarily improve forest conservation. In addition, we find that, whether the North's budget constraint is binding or not, the forestry countries' welfare cannot be unconditionally improved under the subsidy program compared to the optimal control or *laisser-faire* scenario. This implies that some conditions have to be taken into consideration to guarantee the implementation of this subsidy program secured by the participation of both players.

The rest of the essay is organized as follows. Section 2 is devoted to the presentation of the model and the scenarios to be analyzed. Section 3 provides the solution of the optimal control problem of the South when the North does not provide any support for rainforest conservation. Section 4 characterizes an open-loop Stackelberg equilibrium of a differential game in which the North is the leader and the South is the follower. Section 5 compares the two scenarios in terms of rainforest size and the South's welfare. Section 6 concludes.

2. The model and scenarios

2.1 The model

Based on the original model proposed by Ehui et al. (1990), Van Soest and Lensink (2000) considered a model involving two agents, an aid recipient (or South) and a donor community (or North) where the former optimizes its stream of discounted revenues from forest exploitation (e.g. timber production), agriculture activity and aid from the North over an infinite horizon. The North's utility depends on the size of the rainforest and on the transfers given to the South. We adopt here a simplified version of their model.

Let $D(t)$ denote the rate of deforestation in the South and $F(t)$ be the size at $t \in [0, T]$ of the rainforest under consideration by the North. The planning horizon T is interpreted here as the terminal date of the aid program set by the North. The forestry revenues are equal to the timber production $q(t)$ times the prevailing price $P(t)$. Assuming that there are n valuable stems per unit of land, the quantity of timber produced is equal to n times $D(t)$. For convenience n is normalized to unity such that the timber price $P(t)$ represents the value of all commercially valuable timber per unit of land and hence $q(t) = D(t)$.[1] The (inverse) demand function is assumed, following a long tradition in economics literature, linear and given by

$$P(t) = \bar{P} - \theta D(t),$$

[1]The production of timber could actually be done by two methods, namely selective logging and clear-cutting methods. The former means that only a fraction of timber is extracted whereas the second extracts all commercially valuable timber and consequently the land is converted to agricultural use. In their model, Van Soest and Lensink (2000) do consider both methods. Here we are neglecting the first method mainly for mathematical tractability.

where $\bar{P}$ is the maximal market price obtained when $D(t)$ tends towards zero and θ is a positive parameter. Hence revenue from forest exploitation is given by $P(t)D(t)$.

Given this deforestation activity, the rainforest evolves according to the following differential equation typical in a non renewable resource context

$$\dot{F}(t) = -D(t); \quad F(0) = F_0, \tag{1}$$

where F_0 denotes the initial size of the rainforest.

The agricultural net revenue depends on the size of land under cultivation, $F_0 - F(t)$, and the monetary yield per unit of land which in turn depends on agricultural price $\bar{P}_A$ and land productivity $\bar{Z}$. The total revenue from agriculture is thus given by $\bar{P}_A\bar{Z}[F_0 - F(t)]$. Here we assume that $\bar{Z}$ is constant which we do believe is not a severe assumption since we are dealing with a (short run) finite horizon. To save on notation, let us denote the monetary yield per unit of land by $Y = \bar{P}_A\bar{Z}$.

An other possible source of revenue for the South is the financial transfer from the North. Denote by $s(t)$ the subsidy rate controlled by the North. As stated previously, we want to have the transfer $S(t)$ at $t \in [0,T]$ dependent (negatively) on deforestation rate and (positively) on the forest size. This can be achieved in a variety of manners and we choose the rather simple specification

$$S(t) = \max\{0, s(t)(F(t) - vD(t))\}, \tag{2}$$

where v is a positive constant. Note that we concentrate in the sequel on the interesting case where the transfer is non-negative, otherwise there is no game. The positivity of $S(t)$ can be achieved for instance by assuming that the "penalty" parameter v is not too high, meaning that the North does not penalize too much the deforestation activities in the South.

Combining the revenues from forest exploitation, agriculture activities and from the North aid program, the total revenue function of the South is thus

$$R(t) = \left(\bar{P} - \theta D(t)\right) D(t) + Y(F_0 - F(t)) + s(t)(F(t) - vD(t)). \tag{3}$$

This function shows clearly that there is a trade-off between deforestation and conservation of the rainforest. Assuming that the South aims at maximizing its stream of revenues over the duration of the aid program, its optimization problem thus reads

$$\begin{aligned} W &= \max_{\{D(t)\}} \int_0^T R(t)\,dt + \phi F(T) \qquad (4) \\ s.t. &: (1) - (3), \end{aligned}$$

where $\phi F(T)$ is the salvage value for the South of the available forest at the terminal date. The linear form here is assumed for simplicity. The extreme case is when ϕ equals zero, which could happen if the revenues of deforestation are so large that the South applies a policy of after-me-the-deluge and by the end of the planning horizon depletes completely the forest.

Turning now to the North optimization problem, we assume that its objective is to maximize the size of the rainforest by the terminal date of its aid program. As mentioned before, this program is endowed with a total budget that we denote B. The budget constraint is obviously

$$\int_0^T S(t)\,dt \leq B.$$

This isoperimetric constraint can be written equivalently as following

$$\dot{y}(t) = S(t); \quad y(0) = 0; \quad y(T) \leq B, \tag{5}$$

where $\dot{y}(t)$ represents the spending rate at t and $y(t)$ is the cumulative spending from the start of the program till time t.

Formally, the optimization problem of the North is thus

$$\begin{aligned} & \max_{\{s(t)\}} F(T) \\ s.t. \quad : \quad & (1) - (2) \text{ and } (5). \end{aligned}$$

To recapitulate, we have defined a two-player non-zero-sum differential game where the South controls the deforestation rate and the North the subsidy rate that determines the transfer flowing to the South. The game involves two state variables, the rainforest size in the recipient country and cumulative spending by the North.

Remark 1 *The discount rate is set to zero which seems to be acceptable in any finite-horizon game. On the top of that, environmentalists are arguing that discounting would not be fair from an intergenerational equity perspective. However, all our results can be easily reproduced with a positive discount rate.*

2.2 The scenarios

To assess the efficiency of the aid program (to which we shall also refer as the coordinated effort game), we shall compare its results to a benchmark *laisser-faire* case. In such scenario, the North does not provide any aid to the South and the problem becomes an optimal control one where the South optimizes its revenues with $S(t) = 0, \forall t \in [0, T]$.

The efficiency of the coordinated effort game with respect to the *laisser-faire* case will be assessed in both environmental and economic terms. Environmental efficiency is measured by the difference between the two scenarios in the rainforest size inherited at terminal date. If the aid program increases this size with respect to the *laisser-faire* case, then the North would be willing to offer to the South such a program.

Economic efficiency is assessed by the difference in total revenues secured by the South with and without the aid program. The South would be willing to implement the deforestation policy prescribed in the coordinated effort game if it results in an increase in its revenue. When both conditions are met, we shall say that the aid program is implementable. Note that implementability here is synonymous with Pareto improvement with respect to the benchmark case.

We still need to clarify which solution concept and information structure we will adopt in the coordinated game. It seems natural to assume that the donor community plays a leadership role in the implementation of the aid program and we assume hence that the game is played *à la* Stackelberg. Regarding the information structure, we shall retain the tractable open-loop one which can be justified here on the ground that the horizon of the aid program is short. The North will announce its subsidy rate and the South will work out its deforestation policy accordingly. Although in general a feedback information structure is more conceptually appealing than its open-loop counterpart, it is plausible to assume in our context that the North would not put its future credibility in jeopardy by retracting at an intermediate date from what it announced at the initial instant of the game. In a different context, Zaccour (1996) and Jørgensen and Zaccour (1999) suggested subsidy schemes for a government wishing to accelerate the diffusion of a new (green) technology in the society based on open-loop (respectively Nash and Stackelberg) information structure arguing that it does not seem empirically plausible that the government changes later on the announced program at initial date. They provide examples where the program remained as when initiated during its whole duration. Note further that the transfer function we suggest depends on the state which confers an adaptability flavor to it.

3. The *laisser-faire* scenario

Here the North does not play and the South solves the optimal control problem in (4) with $S(t) = 0, \forall t \in [0, T]$. We omit from now on the time argument when no confusion may arise. The following proposition provides the optimal solution to the South problem where the superscript L stands for *Laisser-faire.*

Proposition 2 *Assuming an interior solution, the laisser-faire optimal control, state and costate are given by*

$$D^L(t) = \frac{\bar{P} - \phi + Y(T-t)}{2\theta}, \tag{6}$$

$$F^L(t) = F_0 + \frac{Y}{4\theta}t^2 - \frac{\bar{P} - \phi + YT}{2\theta}t, \tag{7}$$

$$\lambda^L(t) = -Y(T-t) + \phi. \tag{8}$$

The South welfare is given by

$$W^L = \frac{Y^2T^2 + 3(\bar{P} - \phi)\left(\bar{P} - \phi + YT\right)}{12\theta}T + \phi F_0. \tag{9}$$

Proof. The Hamiltonian of the South optimal control problem is

$$H^L(F, D, \lambda) = \left(\bar{P} - \theta D\right) D + Y(F_0 - F) - \lambda D,$$

where λ denotes the aid recipient's costate variable associated with the forest stock. The first order conditions are given by:

$$\begin{aligned} H^L_D &= \bar{P} - 2\theta D - \lambda = 0, \\ \dot{F} &= H^L_\lambda = -D < 0; \qquad F(0) = F_0, \\ \dot{\lambda} &= -H^L_F = Y > 0; \quad \lambda(T) = \phi. \end{aligned} \tag{10}$$

The costate differential equation has as its solution

$$\lambda^L(t) = -Y(T-t) + \phi.$$

It is easy then to get the optimal control $D^L(t)$ and the resulting state trajectory. Inserting these values in (4) and integrating leads to the optimal welfare W^L. ■

The results in the above proposition are intuitive. Indeed, it is readily seen from the optimal conditions that the deforestation policy satisfies the familiar rule of marginal revenue from deforestation $(\bar{P} - 2\theta D)$ must equal its marginal cost, given here by the costate λ. Conserving an additional unit of rainforest has two impacts on South*'s* performance. One is positive and is measured by the marginal value at terminal time of rainforest, and the second one corresponds to the loss in agriculture revenues given by the monetary yield by unit of land Y times the remaining time. Furthermore, $D^L(t)$ is an increasing function in the prevailing timber price (since it is decreasing in the slope of the demand law for timber (θ) and increasing in the intercept $(\bar{P})$). In addition, the dynamics of the shadow price (equation (10)) indicates that it is increasing at a constant positive rate equal to the agricultural revenue per unit of land (Y) and reaches its maximum at terminal date where it is equal to ϕ.

Remark 3 *The necessary conditions are also sufficient. Indeed, the Hamiltonian is linear in the state and* $H^L_{DD} = -2\theta < 0$.

Remark 4 *In Proposition 1 we have assumed an interior solution. Since* $\dot{F}^L = -D^L$*, a sufficient condition to have* $F^L(t) > 0, \forall t \in [0,T]$ *is* $F^L(T) > 0$*. Evaluating (7) at* T *and arranging terms, we have*

$$F^L(T) = -\frac{Y}{4\theta}T^2 - \frac{\bar{P} - \phi}{2\theta}T + F_0,$$

which is positive if and only if [2]

$$F_0 \geq \tilde{F}^L = \frac{YT + 2\left(\bar{P} - \phi\right)}{4\theta} T. \tag{11}$$

4. The coordinated Effort Scenario

In this scenario the North (donor community) participates in the conservation effort of the rainforest by compensating the South for its loss of revenues through a subsidy program.

The sequence of events is as follows. The North as the leader in the Stackelberg game moves first and proposes to the South a financial transfer given by (2). The South (follower) optimizes its objective taking into account the leader's announcement and determines its deforestation rate. The main difference between this scenario and the previous one is that the South's trade-off between deforestation and conservation must take into account the additional resource provided by the North as a contribution to conservation effort.

To determine an open-loop Stackelberg equilibrium, we first solve the South problem, that is

$$\begin{aligned} W &= \max_{\{D(t)\}} \int_0^T \left[\left(\bar{P} - \theta D\right) D + Y\left(F_0 - F\right) + S\left(t\right)\right] dt + \phi F\left(T\right), \\ \text{s.t.:} \quad \dot{F}\left(t\right) &= -D\left(t\right); \quad F(0) = F_0, \\ S\left(t\right) &= \max\left\{0, s(t)\left(F\left(t\right) - vD\left(t\right)\right)\right\}. \end{aligned}$$

The South's Hamiltonian of the problem is

$$H_f^C = -\theta D^2 + \left(\bar{P} - sv - \lambda\right) D + \left(s - Y\right) F + Y F_0,$$

[2] If condition (11) is not satisfied, then the forest stock becomes null before the end of the program is reached.

where λ denotes the South's costate variable associated with the forest stock. The superscript C stands for "$c$oordinated scenario" and the subscript f stands for "$f$ollower".

Assuming an interior solution, first order conditions are

$$\begin{aligned} D &= -\frac{1}{2\theta}\left(\lambda + vs - \bar{P}\right), & (12)\\ \dot{F} &= \frac{1}{2\theta}\left(\lambda + vs - \bar{P}\right); \quad F(0) = F_0, & (13)\\ \dot{\lambda} &= Y - s; \qquad\qquad \lambda(T) = \phi. & (14) \end{aligned}$$

As in the optimal control scenario the deforestation rate is decreasing with respect to the shadow price of the forest stock and increasing with the market timber price. Note that D, as expected, depends negatively on the subsidy rate (see equation (12)). The direction of variation of the shadow price of the rainforest (see (14)) depends on the difference between the per unit of land yield from agriculture and the per unit revenue (subsidy) from land conservation. Recall that in the *laisser-faire* scenario we had $\dot{\lambda}^L = Y$.

As a Stackelberg leader, the North incorporates the last two equations in its optimization program, which reads

$$\max_{\{s(t)\}} F(T),$$

$$\text{s.t.:} \quad \begin{aligned} \dot{y} &= s\left(F + \frac{v}{2\theta}\left(\lambda + vs - \bar{P}\right)\right), \qquad y(0) = 0, \quad y(T) \leq B,\\ \dot{F} &= \frac{1}{2\theta}\left(\lambda + vs - \bar{P}\right), \qquad F(0) = F_0,\\ \dot{\lambda} &= Y - s, \qquad \lambda(T) = \phi. \end{aligned}$$

where μ denotes the costate of cumulative expenditures of the North y, η the North's shadow price of the forest stock F and ψ the costate of the South's shadow price of the forest stock.

Let H_l^C be the leader's Hamiltonian which can be written as:

$$H_l^C = \frac{v^2}{2\theta}\mu s^2 - \frac{\bar{P}v}{2\theta}\mu s + \mu s F - \psi s + \frac{v}{2\theta}\eta s + \frac{v}{2\theta}\mu\lambda s + Y\psi - \frac{\bar{P}}{2\theta}\eta + \frac{1}{2\theta}\eta\lambda.$$

The Lagrangian associated with this problem is

$$L_l^C = H_l^C + \kappa\left(B - y\left(T\right)\right),$$

where κ is the multiplier associated with the budget constraint at final time T.

The first order conditions of the leader's optimization problem are:

$$\left(H_l^C\right)_s = \frac{v^2}{\theta}\mu s - \frac{\bar{P}v}{2\theta}\mu + \mu F - \psi + \frac{v}{2\theta}\eta + \frac{v}{2\theta}\mu\lambda = 0. \tag{15}$$

$$\begin{aligned}
\dot{\mu} &= -(H_l^C)_y = 0, & \mu\left(T\right) &= -\kappa, & (16)\\
\dot{\eta} &= -(H_l^C)_F = -\mu s, & \eta\left(T\right) &= 1, & (17)\\
\dot{\psi} &= -(H_l^C)_\lambda = -\frac{1}{2\theta}(v\mu s + \eta), & \psi\left(0\right) &= 0, & (18)\\
\dot{y} &= (H_l^C)_\mu = \frac{s}{2\theta}\left(2\theta F + v^2 s + v\lambda - v\bar{P}\right), & y(0) &= 0, & (19)\\
\dot{F} &= (H_l^C)_\eta = \frac{1}{2\theta}\left(\lambda + vs - \bar{P}\right), & F\left(0\right) &= F_0, & (20)\\
\dot{\lambda} &= (H_l^C)_\psi = Y - s, & \lambda\left(T\right) &= \phi, & (21)
\end{aligned}$$

$$\kappa(B - y(T)) = 0, \quad \kappa \geq 0, \quad y\left(T\right) \leq B. \tag{22}$$

The complementarity terminal condition on the budget constraint in (22) requires the study of two different cases. First, we analyze the case where the North does not spend all its available budget by the end of the program. Second, we look at the case where the budget constraint is "binding".

Case 1. $B > y\left(T\right)$

In this case, the North doesn't spend all its available budget. This means that the budget constraint is not binding and that the multiplier κ is equal to zero, which from (16) implies that the costate variable μ is also equal to zero over all the horizon. From the necessary condition (15) it is easy to see that the problem becomes bang-bang in the control variable s. To give sense to the optimization problem, we assume that the transfer rate s is lower bounded by zero and upper bounded by $\bar{s}$. Then, the optimal control policy can be described by:

$$s^{C_1} = \begin{cases} \bar{s} & \text{if} \quad -\psi + \frac{v}{2\theta}\eta > 0, \\ \hat{s} & \text{if} \quad -\psi + \frac{v}{2\theta}\eta = 0, \quad \hat{s} \in [0, \bar{s}], \\ 0 & \text{if} \quad -\psi + \frac{v}{2\theta}\eta < 0, \end{cases}$$

where the superscript C_1 refers to case 1 of the coordinated scenario.

The next proposition shows that the North's optimal policy is to subsidize the South at the maximum rate for all t.

Proposition 5 $s^{C_1}(t) = \bar{s}$, $\forall t \in [0, T]$.

Proof. Since μ is identically null, equation (17) implies that $\eta(t) = 1, \forall t \in [0, T]$. Therefore, integrating (18), we obtain $\psi(t) = -\frac{1}{2\theta}t$, $\forall t \in [0, T]$, which is always negative. Since $-\psi + \frac{v}{2\theta}\eta = \frac{1}{2\theta}t + \frac{v}{2\theta} > 0$, $\forall t \in [0, T]$, then we obtain $s^{C_1}(t) = \bar{s}$, $\forall t \in [0, T]$. ■

The interpretation of the above proposition is simple: since the subsidy is beneficial to the North's objective and a budget is anyway available and not tight, then clearly the optimal policy is to support maximally the conservation effort of the South.

Proposition 6 *The maximum subsidy rate $\bar{s}$ that the North will offer constantly to the South during the aid program period, if he is spending $y(T)$ an amount strictly* *inferior to the total budget B, satisfies:* $\bar{s} \in (0, \bar{s}_2)$, *where*

$$\bar{s}_2 = \frac{-C_2 + \sqrt{C_2^2 - 4C_1C_3}}{2C_1} > 0,$$

and C_1, C_2 *and* C_3 *are constants given by*

$$\begin{aligned} C_1 &= T\left[v(T+v) + \frac{1}{3}T^2\right], \\ C_2 &= T\left[2\theta F_0 - YT\left(\frac{v}{2} + \frac{T}{3}\right) - (\bar{P} - \phi)\left(v + \frac{T}{2}\right)\right], \\ C_3 &= -2\theta B. \end{aligned} \tag{23}$$

Proof. Note that $\bar{s}$ is the maximum subsidy rate that the North will offer constantly to the South during the program period, if he is spending $y(T)$ an amount strictly inferior to the total budget B. This means that the subsidy rate $\bar{s}$ must be such that the following inequality is satisfied:

$$\int_0^T \bar{s}[F(t) - vD(t)]dt = y(T) < B. \tag{24}$$

Using the budget constraint (24), we can solve for the maximum subsidy rate $\bar{s}$ after substituting $F(t)$ and $D(t)$ by their respective optimal values given by equations (27) and (29).

There is a positive transfer and a North-South agreement, if the following expression is positive

$$F(t) - vD(t) = F_0 - \frac{1}{2\theta}(t+v)[(Y-\bar{s})T + \bar{P} - \phi - \bar{s}v] + \frac{Y - \bar{s}}{4\theta}t(t+2v). \tag{25}$$

In this case, the transfer rate is equal to $\bar{s}$, and this rate has to fulfill the following budget constraint

$$\int_0^T S(t)\,dt \le B = \int_0^T \bar{s}(F(t) - vD(t))\,dt \le B.$$

After substituting $(F(t) - vD(t))$ by its expression given in (25) and the computation of the second integral of the above inequality, the latter can be written as a second degree inequality with respect to $\bar{s}$:

$$T\left[v(T+v)+\frac{1}{3}T^2\right]\bar{s}^2+T\left[2\theta F_0-YT\left(\frac{v}{2}+\frac{T}{3}\right)-(\bar{P}-\phi)\left(v+\frac{T}{2}\right)\right]\bar{s}-2\theta B\leq 0,$$

which can also be written as

$$C_1\bar{s}^2 + C_2\bar{s} + C_3 \leq 0, \tag{26}$$

where constants C_1, C_2 and C_3 are given in (23).

It is obvious that $C_1 > 0, C_3 < 0$. We can then solve for $\bar{s}$ in (26), even if the sign of C_2 is unknown. If we denote by $\bar{s}_1, \bar{s}_2$ the two possible solutions for the equation

$$C_1\bar{s}^2 + C_2\bar{s} + C_3 = 0,$$

we have :

$$\begin{aligned}\bar{s}_1 &= \frac{-C_2 - \sqrt{C_2^2 - 4C_1C_3}}{2C_1} < 0,\\ \bar{s}_2 &= \frac{-C_2 + \sqrt{C_2^2 - 4C_1C_3}}{2C_1} > 0.\end{aligned}$$

This means that the budget constraint as described in the inequality (26) is satisfied if and only if

$$\bar{s} \in (0, \bar{s}_2).$$

■

Given the optimal subsidy rate, integrating equations (19), (20) and (21) we get:

$$F^{C_1}(t) = F_0 + \frac{Y-\bar{s}}{4\theta}t^2 - \frac{\bar{P}-\phi-\bar{s}v+(Y-\bar{s})T}{2\theta}t, \tag{27}$$

$$\lambda^{C_1}(t) = -(Y-\bar{s})(T-t)+\phi, \tag{28}$$

$$y^{C_1}(t) = \frac{\bar{s}t}{2\theta}\Big[(Y-\bar{s})\frac{t^2}{6} - \left(\bar{P}-\phi-vY+T((Y-\bar{s})\right)\frac{t}{2} + 2F_0\theta - v\left(\bar{P}-\phi-v\bar{s}+(Y-\bar{s})T\right)\Big].$$

Finally, replacing the optimal paths of the North's costate variable in equation (12), the optimal deforestation rate path is completely determined:

$$D^{C_1}(t) = \frac{Y-\bar{s}}{2\theta}(T-t) + \frac{\bar{P}-\phi-\bar{s}v}{2\theta}. \tag{29}$$

We can then obtain the South's welfare function:

$$W^{C_1} = \frac{(Y-\bar{s})\left[(Y-\bar{s})T+3\left(\bar{P}-\phi-\bar{s}v\right)\right]T+3\left(\bar{P}-\phi-\bar{s}v\right)^2}{12\theta}T + F_0(\phi+\bar{s}T). \tag{30}$$

Remark 7 *In the above proposition we have assumed an interior solution. To guarantee that $F^{C_1}(t) \geq 0, \forall t \in [0,T]$, it suffices to have $F^{C_1}(T)$ positive. Since*

$$F^{C_1}(T) = F_0 - \frac{Y-\bar{s}}{4\theta}T^2 - \frac{\bar{P}-\phi-\bar{s}v}{2\theta}T,$$

then $F^{C_1}(T)$ is positive if

$$F_0 \geq \tilde{F}^{C_1} = \frac{(Y-\bar{s})T+2\left(\bar{P}-\phi-\bar{s}v\right)}{4\theta}T.$$

Note that the last condition is always satisfied given condition (11), since $\tilde{F}^L \geq \tilde{F}^{C_1}$.

The deforestation rate is decreasing with respect to the subsidy rate, as expected. The difference with the former scenario is that the subsidy rate affects positively the South's shadow price of the forest stock and negatively the deforestation rate. This implies that the subsidy received from the North increases the South's marginal evaluation of the forest stock, counteracting the effect of the agricultural revenue per unit of land which helps to reduce the deforestation rate and to increase the South shadow price by reducing the opportunity cost of conserving the forest stock. This can be easily seen by comparing equations (29) and (28) respectively to equations (6) and (8), where the main difference is the presence of the subsidy rate.

Finally, it is straightforward to see that the welfare is increasing with the per unit of land agriculture revenue, the agricultural product price, the initial size of the forest stock, the final horizon time and the prevailing timber price. It is however decreasing in the punishing coefficient related to the deforestation rate v in the subsidy function, implying an active incentive for the South to preserve the forest stock. The welfare function also increases as the maximum subsidy rate $\bar{s}$ or the constant ϕ in the salvage value function increases. These results are fairly intuitive.

Remark 8 *Note that $\bar{s}$ is the maximum subsidy rate that the North will offer constantly to the South during the program period. If it happens that the total amount spent $y(T)$ is strictly inferior to the total budget B, then the subsidy rate $\bar{s}$ should be increased since the North can achieve a better result with the same budget. Thus, the most economically interesting case would be the one where there is a binding constraint, i.e., the whole budget is spent and $\bar{s} = \bar{s}_2$.*

Case 2. $y(T) = B$

In this case we study the case where the North faces a binding budget constraint and, consequently, has to spends all its available budget.

From (22) we have a positive Lagrangian multiplier κ. Further, from (16), we have

$$\mu(t) = -\kappa, \ \forall t \in [0, T].$$

Therefore, the North's costate variable, μ, associated with the accumulated subsidy spent by this region, y, takes always a constant and negative value. A constant value means that the marginal contribution to the objective of each dollar spent is the same. Being non-positive simply reflects the fact that the more money which has been spent by time t, the less is left in the budget. Recall that the budget is given here. Using this expression for variable μ, the optimal condition (15) gives the following expression for the subsidy rate:

$$s^{C_2} = -\frac{\theta}{v^2}F - \frac{1}{2v}\lambda - \frac{\theta}{v^2}\frac{\psi}{\kappa} + \frac{1}{2v}\frac{\eta}{\kappa} + \frac{\bar{P}}{2v}, \tag{31}$$

where the superscript C_2 stands for case 2 in the Coordinated scenario.

The North's optimal subsidy rate is decreasing with the forest stock, implying that the North's willingness to pay for the forest stock conservation is higher when the forest stock becomes smaller. The subsidy rate is increasing (respectively decreasing) with respect to the North's forest shadow price (respectively the South's). This is an expected result since the North will increase its subsidy rate as its marginal evaluation of the forest increases. On the contrary it will increase the subsidy rate if the South's marginal evaluation of the forest stock decreases because it implies a higher deforestation rate. In other words, the lower is the South evaluation of the forest stock, the more it will tend to deforest and the larger the subsidy rate offered by the North will be. Finally we note that the subsidy rate is increasing in the timber market price: The North should increase its compensation (subsidy) to prevent that the South, induced by higher potential forestry revenues, increases its deforestation rate.

Contrary to the case in which the North does not spend all its budget at the end of the time horizon, now the dynamic system of the optimal conditions is coupled and cannot be solved recursively as we have previously done. Integrating the system of differential equations and after some manipulations, we obtain the final expressions of the optimal time paths:

$$\begin{aligned} F^{C_2}(t) &= -\frac{Y}{8\theta}(2T-t)t + \frac{1-\kappa\left(\bar{P}-\phi\right)}{4\theta\kappa}t + \frac{F_0}{2}\left(e^{-\frac{1}{v}t}+1\right), \qquad (32)\\ \eta^{C_2}(t) &= -\frac{\kappa Y}{2}(T-t) - \kappa\frac{\bar{P}-\phi-vY}{2}\left(1-e^{-\frac{1}{v}(T-t)}\right)\\ &\quad +\frac{1}{2}\left(1+e^{-\frac{1}{v}(T-t)}\right) + \frac{\kappa\theta F_0}{2v}\left(e^{-\frac{1}{v}t}-e^{-\frac{1}{v}(2T-t)}\right),\\ \lambda^{C_2}(t) &= -\frac{Y}{2}(T-t) - \frac{1+\kappa(\bar{P}-\phi-vY)}{2\kappa}e^{-\frac{1}{v}(T-t)}\\ &\quad -\frac{\theta F_0}{2v}\left(e^{-\frac{1}{v}t}-e^{-\frac{1}{v}(2T-t)}\right) + \frac{1+\kappa\left(\bar{P}+\phi-vY\right)}{2\kappa},\\ \psi^{C_2}(t) &= \frac{\kappa Y}{8\theta}(2T-t)t - \frac{1-\kappa(\bar{P}-\phi)}{4\theta}t + \frac{\kappa F_0}{2}\left(e^{-\frac{1}{v}t}-1\right). \end{aligned}$$

Replacing these expressions in the South's optimal deforestation rate and the North's optimal transfer rate we obtain

$$D^{C_2}(t) = \frac{Y}{4\theta}(T-t) + \frac{F_0}{2v}e^{-\frac{1}{v}t} + \frac{\kappa\left(\bar{P}-\phi\right)-1}{4\theta\kappa}, \qquad (33)$$

$$s^{C_2}(t) = \frac{1+\kappa\left(\bar{P}-\phi\right)}{2\kappa v}e^{-\frac{1}{v}(T-t)} + \frac{Y}{2}\left(1-e^{-\frac{1}{v}(T-t)}\right) - \frac{\theta F_0}{2v^2}\left(e^{-\frac{1}{v}t}+e^{-\frac{1}{v}(2T-t)}\right). \qquad (34)$$

Finally replacing the corresponding expressions in the South's welfare function we get

$$\begin{aligned} W^{C_2} &= \frac{\left[\kappa YT+3\left(\kappa\left(\bar{P}-\phi\right)-1\right)\right]YT^2}{48\theta\kappa} + \frac{\left[\left(\kappa\left(\bar{P}-\phi\right)-1\right)^2+12\kappa^2\theta YF_0\right]T}{16\kappa^2\theta}\\ &+\frac{\phi F_0}{2}\left(1+e^{-\frac{1}{v}T}\right)\frac{3\theta F_0^2}{8v}\left(1-e^{-\frac{2}{v}T}\right) + \frac{\left[\kappa\left(3\bar{P}-\phi-3vY\right)+1\right]F_0}{4\kappa}\left(1-e^{-\frac{1}{v}T}\right). \qquad (35) \end{aligned}$$

Remark 9 *In the above proposition we have assumed an interior solution. To guarantee that* $F^{C_2}(t) \geq 0, \forall t \in [0,T]$*, it suffices to have* $F^{C_2}(T)$ *positive. Since*

$$F^{C_2}(T) = -\frac{Y}{8\theta}T^2 + \frac{1-\kappa\left(\bar{P}-\phi\right)}{4\theta\kappa}T + \frac{F_0}{2}\left(e^{-\frac{1}{v}T}+1\right),$$

then $F^{C_2}(T)$ *is positive for*

$$F_0 > \frac{[\kappa\left(YT + 2\left(\bar{P}-\phi\right)\right) - 2]T}{4\theta\kappa\left(e^{-\frac{1}{v}T}+1\right)}.$$

Differentiating with respect to the time variable the deforestation rate shows that it decreases over time, which means that as the forest stock decreases, the South applies a lower deforestation rate. Sensitivity analysis of $D^{C_2}(t)$ allows to obtain the following intuitive results, i.e. the deforestation rate is

- increasing with respect to monetary yield from agricultural and to timber's market price.
- decreasing with respect to the punishing coefficient v in the subsidy formula function (showing that the latter is playing its expected role of discouraging deforestation). It is also decreasing with respect to the salvage value the $South$ attributes to the terminal rainforest stock.

Regarding the subsidy rate, it is easy to check that it is increasing in timber price and in agricultural yield, showing actually that the higher is the potential loss of the South from conservation, the higher the cost for the North to achieve its objective. The subsidy rate is decreasing in the initial forest size showing that the less the rainforest is vulnerable, the less is needed to be done by the North.

Finally, note that the impact of a change in the key parameters, one at a time, on the welfare of the South is generally ambiguous.

5. Comparison of scenarios

The aim of this section is to assess the implementation of the coordinated effort scenario. As stated previously, the implementability of the coordinated scenario requires that it improves the South's welfare and the rainforest at terminal date of the subsidy program, which corresponds to the North's objective.

The following proposition provides the result regarding the rainforest.

Proposition 10 *(i)* $F^{C_1}(T) > F^L(T)$.

(ii) *If* $F_0 < \bar{F}_{IN}^{C_2} = \frac{\kappa YT+2(1+\kappa(\bar{P}-\phi))}{4\theta\kappa(1-e^{-\frac{1}{v}T})}T$, *then* $F^{C_2}(T) > F^L(T)$.

Proof. From (7), (27) and (32), we get the terminal rainforest stocks

$$\begin{aligned} F^L(T) &= \frac{1}{4\theta}\left[4\theta F_0 - YT^2 - 2\left(\bar{P}-\phi\right)T\right], \\ F^{C_1}(T) &= \frac{1}{4\theta}\left[4\theta F_0 - (Y-\bar{s})\,T^2 - 2\left(\bar{P}-\phi\right)T + 2\bar{s}vT\right], \\ F^{C_2}(T) &= \frac{1}{8\theta}\left[4\theta F_0\left(e^{-\frac{1}{v}T}+1\right) - YT^2 + \frac{2T}{\kappa}\left(1-\kappa\left(\bar{P}-\phi\right)\right)\right]. \end{aligned}$$

Clearly,

$$\begin{aligned} F^{C_1}(T) - F^L(T) &= \frac{\bar{s}T}{4\theta}\left(T+2v\right) > 0, \\ F^{C_2}(T) - F^L(T) &= \frac{1}{8\theta}\left[4\theta F_0\left(e^{-\frac{1}{v}T}-1\right) + YT^2 + \frac{2T}{\kappa}(1+\kappa(\bar{P}-\phi))\right]. \end{aligned}$$

Last expression is positive if and only $F_0 < \bar{F}_{IN}^{C_2}$. ■

The proposition shows that the subsidy program is always achieving its goal regarding forest preservation, if the budget constraint is not stringent. To guarantee the same result when the total budget is tight, a lower bound on the initial rainforest size is needed. Note that both of the above differences ($F^{C_1}(T) - F^L(T)$ and

$F^{C_2}(T) - F^L(T))$ are increasing functions in the penalty parameter v. This shows, again, the relevance of considering a subsidy formula based on both the forest stock and the deforestation level. Further the improvement in the rainforest stock with respect to *laisser-faire* scenario could be assessed by the North in unit of land terms, by computing the ratio $\frac{F^{C_1}(T)-F^L(T)}{y^{C_1}(T)}$ for the case where the total budget is not exhausted and $\frac{F^{C_2}(T)-F^L(T)}{B}$ for the other case.

The above result constitutes a sufficient condition for the North to implement the subsidy program. We may still wonder how the rainforest trajectories compare under the two regimes (*laisser-faire* and coordinated effort).

Proposition 11 *For the satisfaction of the economic efficiency, comparing the rainforest trajectories, we have*

(*i*) $F^{C_1}(t) > F^L(t), \forall\ t \in [0,T]$.

(*ii*) *If* $F_0 < \frac{\kappa Y(2T-t)+2\left(1+\kappa\left(\bar{P}-\phi\right)\right)}{4\theta\kappa\left(1-e^{-\frac{1}{v}t}\right)}t, \forall\ t \in [0,T]$, *then* $F^{C_2}(t) > F^L(t)$, $\forall t \in [0,T]$.

Proof. (i) From (7) and (27) we have

$$F^{C_1}(t) - F^L(t) = \frac{(2T-t)+2v}{4\theta}t\bar{s} > 0, \quad \forall t \in [0,T].$$

(ii) From (7) and (32)

$$F^{C_2}(t) - F^L(t) = \frac{-\kappa Y t^2 + 2\left(1+\kappa\left(\bar{P}-\phi+TY\right)\right)t}{8\theta\kappa} - \frac{\left(1-e^{-\frac{1}{v}t}\right)F_0}{2}$$

which is clearly positive if

$$F_0 < \frac{\kappa Y(2T-t)+2\left(\kappa\left(\bar{P}-\phi\right)+1\right)}{4\theta\kappa\left(1-e^{-\frac{1}{v}t}\right)}t, \quad \forall t \in [0,T].$$

■

The above proposition shows that if the budget constraint is not active, then the coordinated rainforest trajectory is always above its *laisser-faire* counterpart. When the budget constraint is tight, this result is ensured only if certain condition on the initial size of the forest is satisfied.

We will now assess the implementability conditions for the South. Theses conditions compare the welfare, assuming that the follower will indeed implement the *laisser-faire* strategy, which is not guaranteed automatically in our setting. In effect, the South still has the possibility to play the uncoordinated-strategy in the coordination scenario when this latter improves its fate. In this case he will receive a non negative payment (eventually zero transfer) from the North. Thus the following proposition states the result under the assumption that the South is really cooperating (always play the stackelberg solution) in the coordinated effort scenario.

Proposition 12 *The economics efficiency is not trivial: The South's welfare under the different scenarios compare as follows:*

(i) *If the following condition on initial rainforest is satisfied*

$$F_0 \geq \underline{F}_{IS}^{C_1} = \frac{(2Y - \bar{s})\, T^2 + 3\left(\bar{P} - \phi + v\,(Y - \bar{s})\right) T + 3v\left(2\left(\bar{P} - \phi\right) - v\bar{s}\right)}{12\theta},$$

then $W^{C_1} > W^L$.

(ii) *If one of the following conditions on initial rainforest is satisfied*

$$F_0 \leq \bar{F}_{IS}^{C_2} = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \textit{and} \quad c > 0,$$

or

$$\underline{F}_{IS}^{C_2} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \leq F_0 \leq \bar{F}_{IS}^{C_2} \quad \textit{and} \quad b > 0, c < 0, b^2 - 4ac > 0,$$

then $W^{C_2} > W^L$, *where*

$$\begin{aligned} a &= -\frac{3\theta}{8v}(1 - e^{-\frac{2}{v}T}) < 0, \\ b &= \frac{1}{4\kappa}[(1 - e^{-\frac{1}{v}T})(1 + 3\kappa(\bar{P} - \phi - vY)) + 3TY], \\ c &= -\frac{T}{16\theta\kappa^2}[-1 + 2\kappa(\bar{P} - \phi) + 3\kappa^2(\bar{P} - \phi)^2 + TY(TY + 1 + 3\kappa(\bar{P} - \phi))]. \end{aligned} \tag{36}$$

Proof. (i) Use (30) and (9) to compute

$$W^{C_1} - W^L = -\frac{\bar{s}\left(2Y - \bar{s}\right)T^2 + 3\bar{s}\left(\bar{P} - \phi + v\left(Y - \bar{s}\right)\right)T + 3v\bar{s}\left(2\left(\bar{P} - \phi\right) - v\bar{s}\right)}{12\theta}T + \bar{s}F_0T$$

which is positive if $F_0 \geq \underline{F}_{IS}^{C_1}$.

(ii) Use (35) and (9) to compute

$$\begin{aligned} W^{C_2} - W^L = &-\frac{1}{16\theta\kappa^2}[Y\kappa T^2(Y\kappa T + 1 + 3\kappa(\bar{P} - \phi)) + \\ &(3\kappa^2(\bar{P} - \phi)^2 + 2\kappa(\bar{P} - \phi) - 1 - 12F_0Y\theta\kappa^2)T + 16F_0\theta\kappa^2\phi] - \\ &\frac{3\theta}{8v}(1 - e^{-\frac{2}{v}T})F_0^2 + \frac{1}{4\kappa}(1 - e^{-\frac{1}{v}T})F_0(1 + \kappa(3\bar{P} - 3vY - \phi)) + \\ &\frac{1}{2}(1 + e^{-\frac{1}{v}T})F_0\phi. \end{aligned}$$

The above expression can be rewritten as a second-order polynomial in variable F_0:

$$W^{C_2} - W^L = aF_0^2 + bF_0 + c,$$

where the constants a, b and c are given in (36).

Studying the sign of the polynomial, we have different results depending on the sign of constants b and c and on the sign of the discriminant associated to the second-order polynomial: $\Delta = b^2 - 4ac$.

If $c > 0$, then

$$W^{C_2} - W^L > 0 \;\Leftrightarrow F_0 \leq \bar{F}_{IS}^{C_2}.$$

If $c < 0$ and $\Delta < 0$, then always $W^{C_2} - W^L < 0$ for any F_0 positive.

If $c < 0, b < 0$ and $\Delta > 0$, then always $W^{C_2} - W^L < 0$ for any F_0 positive.

If $c < 0, b > 0$ and $\Delta > 0$, we have:

$$W^{C_2} - W^L > 0 \quad \Leftrightarrow \underline{F}_{IS}^{C_2} \leq F_0 \leq \bar{F}_{IS}^{C_2}.$$

■

The main message of the above proposition is that the South will see its welfare improved with respect to *laisser-faire* scenario only if a certain condition on initial state of rainforest is satisfied. In both cases, whether the aid budget is tight or not, the conditions to guarantee such improvement depend on all the model's parameters and does not offer a qualitative insight.

Next corollary summarizes the conditions for joint implementability of the subsidy program for North and South.

Corollary 13 *The subsidy program will be implemented*

(i) When the budget constraint is not tight, if the following condition on initial rainforest is satisfied

$$F_0 > \underline{F}_{IS}^{C_1}.$$

(ii) When the budget constraint is active, if one of the following conditions on initial rainforest is satisfied

$$F_0 < \inf\{\bar{F}_{IN}^{C_2}, \bar{F}_{IS}^{C_2}\} \quad \text{and} \quad c > 0.$$

or

$$\underline{F}_{IS}^{C_2} < F_0 < \inf\{\bar{F}_{IN}^{C_2}, \bar{F}_{IS}^{C_2}\} \quad and \quad b > 0, c < 0, b^2 - 4ac > 0.$$

The previous corollary indicates that in case the North does not spend all its available budget, the joint implementability condition is just that of the South. However, when the North does spend all its budget, the implementability of the subsidy program depends on both players' conditions to join the program. This means that the case where the North is not limited by the budget available (case 1) can reach its objective of forest conservation, in the sense that the forest stock is unambiguously improved in contrast with the case where it is constrained by the budget available (case 2).

6. Conclusion

Using a model representing an economic/ecological interaction of the rainforest, we showed that it is possible for the North to design a subsidy program to help the South in managing this forest. The setting is of a differential game with a leader-follower information structure played over a finite horizon. The results indicate that making the transfer function dependent on the deforestation rate directly in addition to be dependent on the forest stock, has a clear impact on (slowing) deforestation. This result is consistent with the findings in Van Soest and Lensink (2000). In the case of a non effective budget constraint (the budget in not binding as in case 1), we also join the results of previous literature (see for example; Stahler (1996), Mohr (1996), Van soest and Lensink (2000), Martín-Herrán et al. (2002)) stating that funding forest preservation leads to its unambiguous improve.

However, contrary to these previous studies, we showed that in some cases (when the budget constraint is binding as in case 2) additional conditions may be needed to ensure an improvement of forest conservation under the subsidy program. In addition, we found that the forestry countries' welfare cannot be unconditionally improved under the subsidy program compared to the optimal control or *laisser-faire* scenario. This means that some conditions have to be taken into consideration to guarantee the participation of the North and the South in this program, which is not trivial as stated in the previous literature, cited above.

To be able to derive qualitative results and to concentrate on the main point, i.e. the design of a subsidy program, we made some simplifying assumptions. One could extend this work along the following lines.

- A sensitivity analysis on the budget could be done to check if there is a range of values that lead to acceptability of the aid program by the two players. One could also assume that the budget can be replenished by the North: Another extension is to introduce the cost of the budget spent by the North in its objective function. Under this hypothesis the North would look for an optimal trade-off between final forest size and budget spent for the subsidy program.

- The subsidy program is implemented here as part of an open-loop Stackelberg equilibrium. It would be of methodological interest to compare our results to those that would be obtained by defining the game as a cooperative one or by assuming still a non cooperative setting and use so-called incentive equilibrium where the North could condition its subsidy directly to South's action.

ESSAY 2

Incentive Mechanisms to Enforce Sustainable Forest Exploitation

1. Introduction

The global rate of tropical forest destruction is raising general concern. According to the Foods and Agriculture Organization's (FAO) estimates, 53000 square miles of tropical forests were destroyed each year during the eighties. A report by the World Resources Institute (see Matthews (2001)) confirms that the deforestation rate is not slowing but on the contrary continues to be rapid.

The adverse consequences of tropical deforestation on biodiversity conservation and climate change are considerable. In fact, the rate of carbon dioxide (one of the main factors of global warming) released in the atmosphere due to tropical deforestation is approximately 1.6 billion metric tons per year. This constitutes a significant contribution compared to a rate of 6 billion metric tons of carbon emitted from fossil fuel burning (Earth observatory, 2001). Moreover, serious scientific estimates indicate that, on the average, 137 species forms of life (plants and animals) are driven into extinction every day due to habitat destruction (Wilson, 1992).

The main causes of tropical deforestation seem to be the conversion of forested land to agricultural use and, to a lower level, forestry activities (e.g. Amelung and Diehl, 1992; Barbier et al., 1991; Barbier and Burgess, 1997; Kaimowitz and Angelsen, 1999; Southgate et al., 1991; and Southgate, 1990).

It is easily understood that since a country gets revenue from agriculture and forestry activities, the temptation is high to follow a *laissez-faire* policy when it comes to deforestation. In fact, scholars are pointing out that even if the costs of forest preservation are small compared to the large non-economic benefits from doing so, at a domestic level these benefits remain much smaller than the global ones (e.g. Montgomery, 2002; Chomitz and Kumari, 1998; Cline, 1992). Barret (1994-a), Von

Amsberg (1994), Van Soest (1998) and Van Soest and Lensink (2000) argue that in some instances the allocation of forest lands to alternative use may enhance the domestic country's welfare, while decreasing welfare from the global perspective. This confers to deforestation its international externality dimension. One possibility to tackle this problem by rich countries which value the tropical forest is to financially compensate those forestry countries which accept implementation of conservation policies.

Given the dynamic nature of this problem, the literature has adopted differential games and optimal control theory formalisms to design financial transfers from the developed countries (termed the *North*) to forestry developing countries (called the *South*). Barbier and Rauscher (1994), using optimal control theory, consider lump-sum aid donations as an indirect instrument to conserve the forest by reducing the necessity to exploit it. Lump-sum transfers have been however criticized as being a passive instrument to combat deforestation. A more active way to use transfers would be to make the amount of payment conditional to the effort deployed by the recipient country to conserve the tropical forest.

From this perspective, a transfer that consists of paying a fixed price per unit of land conserved was proposed by Stähler (1996) who used an optimal control approach and by Mohr (1996) of a bargaining game framework. These authors raise the point that this kind of dependency between the transfers and the forest size may, however, have adverse effects in the long run. As the international community's willingness to pay is higher when the forested land becomes smaller, the forestry country can use its "market power" to raise the per-unit compensation through a strategic deforestation behavior. Mohr (1996) explained that, in this case, the credibility problem about whether the donor community is indeed "hard nosed" about the fixed compensation can give an incentive to the recipient country to increase its deforestation rate.

Consequently, a transfer payment that penalizes an excessive deforestation rate in addition to compensating for each unit of conserved forested land may be advantageous, especially if we consider that the ecosystem biodiversity is, to a large extent, irreversible. Such a transfer payment, which makes the recipient countries directly confront the results of their land use decisions, was first used in a contract approach by Van Soest and Lensink (2000), and in a differential game approach by Fredj et al. (2004) and Martín-Herrán et al. (2002).

In all these models, the $North$'s main concern is forest preservation, whereas the forestry country's objective is the maximization of the total revenues it can extract from forestry and agricultural activities. In other words, the problem here is essentially that resource managers do not internalize the loss of biodiversity or the loss of an important carbon sink in their objective function. This means that the benefits from tropical forests conservation that represent the concerns of the $North$ (such as cure for cancer and other illnesses, and the mitigation of climate change) are ignored by the domestic forestry countries.;

Consequently, and as it has been proven in the above studies, it should be in the interest of both the $North$ and the $South$ to implement the agreed transfer program. Nevertheless, there is no guarantee that both players would stick to their respective engagements as dictated by the agreed policy. One of the players can have an incentive to deviate from the initial agreement if it knows that the other player will stick to its engagement. The strategies implied by the agreement are then no longer an equilibrium.

This calls for the use of incentive mechanism strategies to counter this eventual problem. The externality problem in several environmental issues has led to the use of incentive mechanism design, e.g. Jørgensen and Zaccour (2001) use incentive equilibrium strategies in the pollution control problem. Originally the idea of incentive equilibrium has been developed and applied to resource management problems by Ehtamo and Hämäläinen (1986, 1989, 1993).

This paper is concerned with the design of incentive strategies mechanisms that can be used by a donor community so that the forestry countries find it optimal to choose in equilibrium a deforestation policy which is sustainable in the long run. We assume that the Northern community has a real concern about forest preservation. Therefore, it can not put its future credibility in jeopardy, by retracting at an intermediate date, from what it announced at the initial instant of the game; especially considering that this environmental agreement is expected to be renewed at the end of the horizon or later on. We, then, exclude the possibility of deviation from this player. Unilateral deviation remains, however, possible from the forestry country as it can be more beneficial for it to receive the compensating transfer without making any effort of forest conservation. To avoid this problem, the $North$ can use an incentive mechanism to force the forestry country to stick to the agreed strategy.

The objective of the Northern community is that, while maximizing its stream of revenues, the forestry country chooses a deforestation rate that is sustainable in the long run. The official definition of sustainable development stated in the World Commission on Environment and Development's report (1987) is "development that meets the needs of the present without compromising the ability of future generations to meet their own needs".

To guarantee the participation of the forestry country in the program, the agreement should ensure at least the same actualized revenues as what the forestry country can have without the developed community's intervention, for the period covered by the agreement. Furthermore, to guarantee the implementation of the agreement once the forestry country accepts to receive the transfer, the $North$ has to make the transfer dependent directly on the forestry country's actions regarding the forest exploitation. The objective of doing so is that the forestry country finds it optimal in the short run equilibrium, to choose a deforestation rate that leads to a sustainable forest stock. This happens when for instance the deforestation rate is equal to the forest's natural regeneration (see, for example, Beltratti, 1995). Our results show that this is possible and can be implemented using an incentive transfer mechanism where the amount of transfers is linear in the forest stock and linear-quadratic in the deforestation rate.

The main contributions of the paper are the following. First, we solve for both the short-run and long-run equilibria. This allows us to calculate the sustainable forest exploitation (calculated for the long-run equilibrium) and to estimate the loss in total welfare the forestry country can bear due to a more sustainable exploitation.

Second, we define a transfer function which guarantees the participation of the recipient country by compensating it for the total loss due to a better forest conservation. Indeed, this transfer designs an incentive mechanism ensuring that the forestry country will respect its engagement regarding a sustainable forest exploitation. To our knowledge this kind of incentive mechanism has never been used previously in the sustainable forest management literature.

In contrast with previous studies we do not need to model explicitly the developed community's utility function which palliates one of the main criticisms addressed in this literature. Moreover, we consider different dynamics of the forest stock as we allow the possibility of regeneration of the resource.

The rest of the paper is organized as follows: Section 2 introduces the model; Section 3 characterizes the optimal control solutions for the forestry country without the intervention of the Northern community, under the finite and infinite planning horizons; Section 4 deals with the design of an incentive mechanism that guarantees the application of the sustainable forest exploitation at each time t, and the distribution of the total transfer payment over time; Section 5 provides a numerical illustration of our results; Finally, section 6 concludes.

2. The model

The original model is proposed for the forestry country by Ehui et al. (1990) as an allocation problem of forested land between forest activities and agricultural use. The objective is to maximize the present value of a utility index $U(\cdot)$, which measures society's satisfaction. The utility index is concavely increasing in the aggregate benefit (π), measuring the societal benefit from agriculture and forestry. The latter depends mainly on the forest size, the deforestation rate and the soil's productivity.

A detailed and sophisticated specification of this model is proposed by Van Soest and Lensink (2000), where the utility index is presented as the total net revenues from agricultural and forest use.

The revenues from forest activities are equal at each time t to the wood production $q(t)$ times the prevailing price of timber $P(t)$. The timber can be extracted using clearfelling or selective logging methods. Under the first method the land is completely deforested and converted to agricultural use. Assuming that there are n commercially valuable stems per unit of land, the quantity of timber supplied at each period of time t is then equal to n times the deforestation rate $D(t)$. Under the selective logging method, only a fraction γ of timber is extracted and the quantity supplied at each period of time t is equal to γ times n multiplied by the size of the forest $F(t)$. The total timber production can be described as follows[1]:

$$q(t) = nD(t) + n\gamma F(t). \tag{1}$$

For convenience n is normalized to unity such that the timber price $P(t)$ represents the value of all commercially valuable timber per unit of land. The (inverse) demand function is assumed, following a long tradition in economics literature, linear and given by

$$P(t) = \bar{P} - \theta q(t), \tag{2}$$

where $\bar{P}$ is the maximal market price obtained when $D(t)$ tends toward zero and θ is a positive parameter.

The agricultural net revenue depends on the size of the land under cultivation $F_0 - F(t)$, where F_0 is the initial size of the forest stock; on the agricultural price which is fixed to $\bar{P}_A$; and on land productivity $Z(t)$. The latter depends positively on the deforestation rate since burning off the forest cover releases nutrients that increase the soil productivity in the short run. It is decreasing in the cumulative deforestation as, in the long run, soil productivity falls because of nutrient depletion and decreases because the increasing distance of forest cover causes the deceleration

[1]In the original specification of the model by Van Soest and Lensink (2000), the timber supply under the selective method is equal to $n\gamma(F(t) - D(t))$. As we are working with a dynamic model we find it more convenient to consider that the size of the forest at any time is equal to the size of land not converted to agriculture use.

of the soil formation:

$$Z(t) = \bar{Z} + \alpha D(t) - \beta [F_0 - F(t)]. \tag{3}$$

Combining the revenues from forest exploitation and agriculture activities the total revenue function of the *South* is thus

$$R(t) = P(t) q(t) + \bar{P}_A Z(t) [F_0 - F(t)]. \tag{4}$$

This function shows clearly that there is a trade-off between deforestation and conservation of tropical forests.

We suppose that the tropical forest evolves according to the following differential equation typical in a renewable resource context[2],

$$\dot{F}(t) = -D(t) + rF(t); \quad F(0) = F_0, \tag{5}$$

where r denotes the natural regeneration rate of the tropical forest.

The forestry country choosing the deforestation rate aims at maximizing its stream of revenues discounted at rate ρ:

$$\int_0^T e^{-\rho t} R(t)\, dt + e^{-\rho T} \Phi(F(T)),$$
$$\text{subject to } (1) - (5),$$

where function Φ is the salvage value function. The time horizon $[0, T]$ may be bounded or unbounded depending on whether T is finite or infinite. In the infinite horizon case, the salvage value function is identically null.

[2] In the model specified by Van Soest and Lensink (2000), the forest dynamics is typical of a non renewable resource with $\dot{F}(t) = -D(t)$.

This dynamic optimization problem belongs to the well known linear-quadratic class. Indeed, the motion equation of the state variable, the forest stock, is linear and the objective function is quadratic in both the state and control variables. The total revenue function of the *South* can be expressed as:

$$R(t) = -a_1 F(t)^2 + a_2 F(t) - a_3 D(t)^2 + a_4 D(t) - a_5 D(t) F(t) + a_6,$$

where

$$a_1 = \beta \bar{P}_A + \theta \gamma^2, \quad a_2 = \gamma \bar{P} + \bar{P}_A \left(2\beta F_0 - \bar{Z}\right), \quad a_3 = \theta,$$
$$a_4 = \bar{P} + \alpha \bar{P}_A F_0, \quad a_5 = 2\theta\gamma + \alpha \bar{P}_A, \quad a_6 = \bar{P}_A F_0 \left(\bar{Z} - \beta F_0\right). \tag{6}$$

This functional form shows how the forest stock and the deforestation rate affect the instantaneous revenues. $a_1 > 0$ (respectively $a_5 > 0$) reflect the decrease in the forest marginal revenues when the forest stock (respectively the deforestation) rate increases. $a_3 > 0$ (respectively $a_5 > 0$) reflect the decrease in the marginal revenues of deforestation when the deforestation rate (respectively the forest stock) increases.

3. Forest exploitation under finite and infinite planning horizons

In this section we derive optimal forest exploitation by the *South* assuming in turn finite and infinite horizons. We make the conjecture that the short-run (finite-horizon) optimization leads to an over extraction as the forestry country does not take into account the long-run impacts of its actions. For this reason, we consider as a counterpart the optimization problem with an infinite horizon, which leads in the long-run equilibrium to a sustainable exploitation with $\dot{F}(t) = 0$, implying that the deforestation rate at equilibrium in the long-run will be equal to the natural regeneration of the forest.

Our objective is to induce the forestry country to choose a sustainable deforestation rate while optimizing in the short-run (or finite horizon). In a first step we derive the solutions of the optimization problems with finite and infinite time horizons. Afterwards, in the next section, we will design the necessary incentive strategies that can be used by a second party, which could be the Northern community as addressed in the previous literature, that will enforce the sustainable forest exploitation in the forestry country.

3.1 Infinite Horizon

The optimization problem in the infinite horizon case reads as follows

$$\max_{\{D(t)\}} \int_0^\infty e^{-\rho t} R(t)\, dt,$$
$$\text{subject to } (1) - (5)\,.$$

The following proposition provides the optimal solution to the *South* problem when an infinite planning horizon is considered. The superscript s stands for *sustainable* solution.

Proposition 1 *Under the conditions*

$$2a_1 - 2a_3 r(\rho - r) - a_5(\rho - 2r) > 0, \tag{7}$$

and

$$a_2 + a_4(r - \rho) > 0, \quad a_4(2a_1 + ra_5) - a_2(a_5 + 2ra_3) > 0. \tag{8}$$

The optimal control, state and costate variables in the infinite horizon scenario which leads to a forest stock path converging in the long-run to the steady-state, F^*, are given by:

$$F^s(t) = F^* + (F_0 - F^*)\, e^{(\rho-\Delta)\frac{t}{2}}, \tag{9}$$

$$\lambda^s(t) = \lambda^* + (F_0 - F^*)\,(a_3(\rho - 2r - \Delta) - a_5)\, e^{(\rho-\Delta)\frac{t}{2}},$$

$$D^s(t) = rF^* - \frac{1}{2}(\rho - \Delta - 2r)(F_0 - F^*)\, e^{(\rho-\Delta)\frac{t}{2}}, \tag{10}$$

where F^* and λ^* represent, respectively, the long-run equilibrium or steady-state of the forest size and its shadow value, given by the following expressions:

$$F^* = \frac{a_2 + a_4(r-\rho)}{2a_1 - 2a_3 r(\rho - r) - a_5(\rho - 2r)}, \tag{11}$$

$$\lambda^* = \frac{-a_2(a_5 + 2ra_3) + a_4(ra_5 + 2a_1)}{2a_1 - 2a_3 r(\rho - r) - a_5(\rho - 2r)}, \tag{12}$$

and $\Delta = \sqrt{\frac{4a_1 + (\rho-2r)[(\rho-2r)a_3 - 2a_5]}{a_3}}$.

The maximized net revenue per period of time is given by

$$R^s(t) = A_1 e^{(\rho-\Delta)t} + A_2 e^{(\rho-\Delta)\frac{t}{2}} + A_3,$$

where constants A_i for $i = 1, 2, 3$ are given in the Appendix.

Proof. The current value Hamiltonian associated with the problem described above is[3]

$$\begin{aligned} H(D,F,\lambda) &= R(D,F) + \lambda \dot{F} \\ &= -a_1F^2 + a_2F - a_3D^2 + a_4D - a_5DF + a_6 + \lambda(-D + rF), \end{aligned}$$

where λ is the costate variable associated with the forest stock F.

[3]For the rest of the paper, the time argument is omitted when no confusion can arise.

An interior solution must satisfy the first order optimal conditions given by

$$\begin{aligned} H_D &= -2a_3 D - a_5 F - \lambda + a_4 = 0, && (13) \\ \dot{F} &= H_\lambda = -D + rF; \quad F(0) = F_0, && (14) \\ \dot{\lambda} &= \rho\lambda - H_F = 2a_1 F + a_5 D + (\rho - r)\lambda - a_2, && (15) \end{aligned}$$

together with the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda(t) F(t) = 0$.

Since $H_{DD} = -2a_3 < 0$ the above conditions are also sufficient for this maximization problem.

Replacing the parameters a_3, a_4 and a_5 by their original values in terms of the model's parameters as defined in (6), and taking into account equation (1) after few manipulations we can rewrite condition (13) as:

$$\lambda = \bar{P} - 2\theta q + \alpha \bar{P}_A (F_0 - F).$$

The second condition (14) simply describes the dynamics of the forest, decreasing at the deforestation rate D, reduced by the natural regeneration of the forest.

From equation (13) we can also extract the expression of D as a function of the state variable F and its shadow value λ:

$$D = -\frac{1}{2a_3}(\lambda - a_4 + a_5 F). \qquad (16)$$

Replacing the latter in (14) and (15) we get the following system of differential equations:

$$\begin{aligned} \dot{F} &= \left(\frac{a_5}{2a_3} + r\right) F + \frac{1}{2a_3}\lambda - \frac{a_4}{2a_3}; \quad F(0) = F_0, \\ \dot{\lambda} &= \left(2a_1 - \frac{1}{2a_3}a_5^2\right) F + \left(\rho - r - \frac{a_5}{2a_3}\right)\lambda + \frac{a_4 a_5}{2a_3} - a_2. \end{aligned}$$

Solving this dynamic system taking into account the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda(t)$, $F(t) = 0$, we have:

$$\begin{aligned} F^s(t) &= F^* + (F_0 - F^*)\, e^{(\rho-\Delta)\frac{t}{2}}, \\ \lambda^s(t) &= \lambda^* + (F_0 - F^*)\,(a_3(\rho - 2r - \Delta) - a_5)\, e^{(\rho-\Delta)\frac{t}{2}}, \end{aligned}$$

where F^* and λ^* represent respectively the long run equilibrium or steady state of the forest size and its shadow value which expressions are given in the statement of the proposition.

Substituting F and λ by their optimal values F^s and λ^s in (16) we then get the optimal path of the deforestation rate:

$$D^s(t) = -\frac{1}{2a_3}(\lambda^* + a_5F^* - a_4) + \frac{4a_1 - a_5(\Delta - 2r + \rho)}{2(a_3(\Delta - 2r + \rho) - a_5)}(F_0 - F^*)\, e^{(\rho-\Delta)\frac{t}{2}},$$

which after some manipulations can be rewritten as in (10).

In order to have the optimal forest stock path converging to the steady-state F^* we need to impose that $\rho - \Delta < 0$. This condition is satisfied if and only if the following inequality applies:

$$2a_1 - 2a_3r(\rho - r) - a_5(\rho - 2r) > 0.$$

This condition guarantees that the term inside the square root in Δ is always positive. Moreover, under this condition the steady-state values of the forest size and its shadow value are positive if the following conditions are satisfied:

$$a_2 + a_4(r - \rho) > 0, \quad a_4(2a_1 + ra_5) - a_2(a_5 + 2ra_3) > 0.$$

Replacing the deforestation and the forest stock by their optimal paths, D^s and F^s, in the objective function, the net revenues from forest exploitation and agriculture use can be written as

$$R^s(t) = A_1 e^{(\rho-\Delta)t} + A_2 e^{(\rho-\Delta)\frac{t}{2}} + A_3,$$

where constants A_i for $i = 1, 2, 3$ are given in the Appendix. ■

Of interest is the interpretation of the costate variable

$$\lambda = \bar{P} - 2\theta q + \alpha \bar{P}_A (F_0 - F).$$

The above expression means that the optimal deforestation rate at each time t should be such that the marginal cost of deforestation (measured by the current shadow value of the forest) equals the net marginal benefit (the sum of the marginal income from selling the timber $\bar{P} - 2\theta q$ and the additional agriculture revenue $\alpha \bar{P}_A (F_0 - F)$).

To interpret the variation over time of λ, we replace in (15) the parameters a_i $(i = 1, 2, 5)$ by their original values given in (6), to obtain, after some arrangements,

$$\dot{\lambda} = (\rho - r)\lambda - \gamma \left[\bar{P} - 2\theta q\right] + \bar{P}_A \left[\bar{Z} + \alpha D - 2\beta (F_0 - F)\right].$$

This condition indicates that the shadow value of the forest increases or decreases at rate $\rho - r$, depending on the sign of $\rho - r$, reduced by the net benefits of conserving an additional unit of land. The latter is equal to the direct marginal revenues of logging it minus the opportunity cost, which is equal to the indirect marginal agriculture revenue that could be earned from conserving this unit of land which indirectly enhances agricultural yield through the productivity effect.

From now on we assume that $\rho - r < 0$, which guarantees the fulfillment of (7) and the first inequality in (8). The values of the model's parameters are considered such that the second inequality in (8) is satisfied.

The asymptotically stable trajectory for the forest stock increases or decreases towards its steady-state depending on the initial size of the forest. Under condition $\rho - r < 0$, it is easy to see that the convergence of the forest shadow value to its steady-state presents the opposite behavior. That is, if the forest stock increases its shadow value decreases and vice versa.

The steady-state of the forest stock equation (11) can also be written as

$$\begin{aligned} F^* &= \frac{\gamma \bar{P} + \left(\bar{P} + \alpha F_0 P_A\right)(r - \rho) + \bar{P}_A \left(2\beta F_0 - \bar{Z}\right)}{2\beta \bar{P}_A + 2\theta\gamma^2 - 2r\theta(\rho - r) - (\rho - 2r)\left(2\theta\gamma + \alpha \bar{P}_A\right)} \\ &= F_0 - \frac{\bar{P}_A \left(\bar{Z} + \alpha r F_0\right) - (r - \rho + \gamma)\left(\bar{P} - 2(r + \gamma)\theta F_0\right)}{2\beta \bar{P}_A + 2\theta\gamma^2 - 2r\theta(\rho - r) - (\rho - 2r)\left(2\theta\gamma + \alpha \bar{P}_A\right)}. \end{aligned}$$

If $\rho - r < 0$, the denominator of the second term in the right hand side is positive. The forest size will be decreasing $(F^* < F_0)$ if and only if the numerator is also positive. The comparative statics shows that a higher forestry revenues induced by a relative increase in the timber price leads to a higher steady-state of the forest stock. This tallies with the corresponding result in Ehui et al. (1990). An increase in the land productivity has a negative effect on the steady-state level of the forest size. The effect of a relative increase in agriculture price is however undetermined and so is the effect of the discount rate.

3.2 Finite Horizon

In a finite-horizon optimization problem, it is customary to add a salvage value to acknowledge that, although the planning horizon is short, something will still happen after the terminal date. Put differently, the salvage value represents the payoff-to-go starting at T. We shall assume for simplicity that this salvage value is linear in the stock of tropical forest, i.e., $\Phi(F(T)) = \phi F(T)$. Clearly this is an approximation since, in the language of dynamic programming, the value function which measures precisely the payoff-to-go of the problem is not linear. A drastic case occurs when ϕ is taken equal to zero, which would correspond to an after-me-the-deluge policy and a complete depletion of the tropical forest.

The optimization problem to be solved is given by

$$\begin{aligned} &\max_{\{D(t)\}} \int_0^T e^{-\rho t} R(t)\,dt + e^{-\rho T}\phi F(T), \\ &\text{subject to } (1)-(5). \end{aligned}$$

The following proposition provides the optimal solution to this problem where the superscript T stands for bounded finite horizon scenario, with terminal time T.

Proposition 2 *Assuming an interior solution, the short-run optimal control, state and costate variables are given by*

$$F^T(t) = F^* + \frac{a_3(\Delta+\rho-2r)-a_5}{a_5^2-4a_1a_3} C_1 e^{(\rho-\Delta)\frac{t}{2}} + \frac{(\rho-2r-\Delta)a_3-a_5}{a_5^2-4a_1a_3} C_2 e^{(\rho+\Delta)\frac{t}{2}}, \tag{17}$$

$$\lambda^T(t) = \lambda^* + C_1 e^{(\rho-\Delta)\frac{t}{2}} + C_2 e^{(\rho+\Delta)\frac{t}{2}}, \tag{18}$$

$$\begin{aligned} D^T(t) = &\frac{1}{2a_3}(a_4-\lambda^*-a_5F^*) + \frac{[(\Delta+\rho-2r)a_5-4a_1]C_1}{2(a_5^2-4a_1a_3)} e^{(\rho-\Delta)\frac{t}{2}} + \\ &\frac{[4a_1-(\rho-2r-\Delta)a_5]C_2}{2(a_5^2-4a_1a_3)} e^{(\rho+\Delta)\frac{t}{2}}. \end{aligned} \tag{19}$$

The maximized net revenue per period t is given by

$$R^T(t) = B_1 e^{t\rho} + B_2 e^{t(\rho+\Delta)} + B_3 e^{(\rho+\Delta)\frac{t}{2}} + B_4 e^{(\rho-\Delta)t} + B_5 e^{(\rho-\Delta)\frac{t}{2}} + B_6.$$

Constants C_1, C_2 and $B_i, i = 1, \ldots, 6$ are given in the Appendix.

Proof. The first order optimal conditions are given by (13)-(15) together with the transversality condition $\lambda(T) = \phi$.

Following the same reasoning as in the infinite horizon scenario the optimal solutions described in the proposition above can be derived.

Inserting the optimal time paths for the control and state variables, D^T and F^T, in the objective function gives the optimal revenue R^T. ■

The results in the above proposition are intuitive. Indeed, it is readily seen from the optimal conditions that the deforestation policy satisfies the familiar rule of marginal revenue from deforestation being equal to its marginal cost, as in the infinite horizon maximization problem. The interpretation of the results stays the same, except that in the finite problem the forest stock path does not converge to the long-run equilibrium where the forest exploitation is sustainable.

4. Incentive strategies

An assumption in our framework is that the *North* or donor community is willing to compensate the forestry country for the loss in welfare it would incur when following the sustainable strategy for as long as it has the insurance that the *South* will indeed implement the desired conservation policy. To design an incentive mechanism which allows to achieve our objective, we use the following *algorithm*:

1. We first determine the total amount of the transfer by computing the difference between the *South*'s payoff under the short-run and long-run (sustainable) scenarios. Given that the support program is assumed to be in place over the interval $[0, T]$, an assumption must be made on how the *South* would value the forest stock at T. We assume that it uses the same salvage function as in the short-run horizon and thus the total amount to be transferred from the *North*

to the $South$ is given by

$$J^T - J^s = \int_0^T e^{-\rho t}\left(R^T(t) - R^s(t)\right)dt + e^{-\rho T}\phi\left(F^T(T) - F^s(T)\right).$$

Note that the above quantity is interpreted as a minimal requirement to induce the $South$ to choose the sustainable conservation policy.

2. We second assume that the $North$ distribute the total amount over time as an incentive mechanism, that is by letting, as in Van Soest & Lensink (2000) and Fredj et al. (2004), the instantaneous transfer be dependent on both the deforestation rate and the forest stock, i.e.,

$$S^I(t) = S(D, F), \tag{20}$$

where the superscript I stands for $Incentive$ mechanism. We assume that $S(D,F)$ is increasing in F and decreasing in D.

Remark 3 *There are many options to define the instantaneous transfer. Indeed, any time-function $S(t)$ satisfying the requirement that the sum of discounted stream of transfers equals the total amount, i.e.,*

$$\int_0^T e^{-\rho t}S(t)dt = J^T - J^s, \tag{21}$$

is a priory a candidate. To illustrate, $S(t)$ can be taken as the difference in instantaneous revenues, reduced by the present value of the average difference in the forest salvage value at the end of the horizon, i.e.,

$$S(t) = \left(R^T(t) - R^s(t)\right) + \frac{e^{-\rho(T-t)}\phi\left[F^T(T) - F^s(T)\right]}{T},$$

Another option is to have a constant transfer over time:

$$\bar{S} = \frac{\rho}{1 - e^{-\rho T}} \left(J^T - J^s \right), \forall t \in [0, T].$$

Although the above two transfers satisfy (21), they do not guarantee that the South will indeed implement the desired sustainable strategy. That is why a transfer program must depend on the deforestation effort and eventually on the size of the forest is needed.

3. We last deal with the determination of a functional form and the parameters' values for the incentive mechanism. Our approach is to redefine the optimization problem of the *South* which includes now the transfers and obtain its deforestation strategy as a *best response* to $S(D, F)$ which can be specified afterwards. The new optimization problem of the *South* which includes now the transfer is

$$\max_{\{D(t)\}} \int_0^T e^{-\rho t} \left(R(t) + S(D, F) \right) dt + e^{-\rho T} \phi F(T),$$
$$\text{subject to } (1) - (5).$$

We now implement our algorithm. The next proposition provides the difference in welfare between the two scenarios.

Proposition 4 *The difference between the forestry country's welfare for the finite and infinite planning time horizons is given by:*

$$\begin{aligned} J^T - J^s = {} & TB_1 + \frac{B_2}{\Delta}\left(e^{\Delta T} - 1\right) + \frac{2B_3}{\Delta - \rho}\left(e^{-(\rho-\Delta)\frac{T}{2}} - 1\right) - \frac{M_1}{\Delta}\left(e^{-\Delta T} - 1\right) - \\ & \frac{2M_2}{\Delta + \rho}\left(e^{-(\rho+\Delta)\frac{T}{2}} - 1\right) + \phi\left[\frac{(\rho - 2r - \Delta)a_3 - a_5}{a_5^2 - 4a_1a_3} C_2 e^{-(\rho-\Delta)\frac{T}{2}} + \right. \\ & \left. \left(\frac{\left[a_3(\Delta + \rho - 2r) - a_5\right] C_1}{a_5^2 - 4a_1a_3} - (F_0 - F^*) \right) e^{-(\rho+\Delta)\frac{T}{2}} \right], \end{aligned} \tag{22}$$

where constants M_1 and M_2 are given in the Appendix.

Proof. The forestry country's welfare for the period $[0,T]$, under the optimal policies of forest exploitation associated to the finite and infinite horizon scenarios are respectively

$$J^T = \int_0^T e^{-\rho t} R^T(t)\, dt + e^{-\rho T} \phi F^T(T), \quad J^s = \int_0^T e^{-\rho t} R^s(t)\, dt + e^{-\rho T} \phi F^s(T).$$

Therefore, the difference is given by:

$$J^T - J^s = \int_0^T e^{-\rho t} \left(R^T(t) - R^s(t)\right) dt + e^{-\rho T} \phi \left(F^T(T) - F^s(T)\right).$$

After some easy computations, the following expression has been derived:

$$R^T(t) - R^s(t) = B_1 e^{t\rho} + B_2 e^{t(\rho+\Delta)} + B_3 e^{(\rho+\Delta)\frac{t}{2}} + M_1 e^{(\rho-\Delta)t} + M_2 e^{(\rho-\Delta)\frac{t}{2}}, \qquad (23)$$

where constants $B_i, i = 1, \ldots 3$ and $M_j, j = 1, 2$ are given in the Appendix.

The difference between the size of the forest stock at time T under the finite and infinite time horizon scenarios is:

$$\begin{aligned} F^T(T) - F^s(T) \quad = \quad & \left(\frac{[a_3(\Delta + \rho - 2r) - a_5]\, C_1}{a_5^2 - 4a_1 a_3} - (F_0 - F^*) \right) e^{(\rho-\Delta)\frac{T}{2}} + \\ & \frac{[a_3(\rho - 2r - \Delta) - a_5]\, C_2}{a_5^2 - 4a_1 a_3} e^{(\rho+\Delta)\frac{T}{2}}. \end{aligned}$$

Thus, the integration of the difference in (23) over the time interval $[0,T]$ and the addition of the term $e^{-\rho T} \phi \left(F^T(T) - F^s(T)\right)$ leads to the expression in (22). ■

The next proposition characterizes the solution of the new optimization problem of the *South*, i.e., the one including the transfers as revenues.

Proposition 5 *Assuming interior solutions, the optimal control, state and costate variables of the optimization problem when the forestry country receives a transfer* *function as in (20) satisfy:*

$$D^I = \frac{1}{2a_3}\left(\frac{\partial S}{\partial D}(D^I, F^I) - a_5 F^I - \lambda^I + a_4\right), \tag{24}$$

$$\dot{F}^I = -D^I + rF^I; \quad F^I(0) = F_0, \tag{25}$$

$$\dot{\lambda}^I = a_5 D^I + 2a_1 F^I + (\rho - r)\lambda^I - \frac{\partial S}{\partial F}(D^I, F^I) - a_2; \quad \lambda^I(T) = \phi. \tag{26}$$

Proof. The present value of the Hamiltonian associated with the new optimization problem is given by:

$$H^I = -a_1 F^2 + a_2 F - a_3 D^2 + a_4 D - a_5 DF + S(D, F) + \lambda^I(-D + rF) + \bar{S} + a_6,$$

where λ^I denotes the new costate variable associated with the forest stock.

Assuming interior solution the necessary conditions of the Maximum Principle of Pontryagin that guarantee the maximization of the new discounted stream of revenues are given by (24)-(26). ■

The incentive mechanism which guarantees the implementation of the sustainable exploitation path is such that F^s, the forest stock path for the infinite horizon optimization problem, given in (9), is the solution of the system (24)-(26).

The functional form of the transfer does not really matter for as long as it fulfills its objective of ensuring implementability of the sustainable strategy. Therefore, easiness of computation becomes an important argument in choosing this form. If we take $S(F, D)$ linear, i.e.,

$$S(D, F) = v(t)\,F(t) - w(t)\,D(t), \tag{27}$$

where $v(t)$ and $w(t)$ are nonnegative functions, then the dynamic system (25)-(26) can be expressed, once the expression of D in (24) has been included, as a linear system in F and λ. The latter can be expressed in matrix form as follows:

$$\begin{bmatrix} \dot{F} \\ \dot{\lambda} \end{bmatrix} = A^I(t) \begin{bmatrix} F \\ \lambda \end{bmatrix} + B^I(t), \tag{28}$$

with boundary conditions

$$F(0) = F_0 \quad \text{and} \quad \lambda(T) = \phi, \tag{29}$$

where both the matrix $A^I(t)$ and the vector $B^I(t)$ depend on the specification and parameters of $S(D,F)$. To have as optimal path for the state variable the sustainable solution, we impose that

$$F^I(t) = F^s(t) = F^* + (F_0 - F^*)\, e^{(\rho-\Delta)\frac{t}{2}},\ \forall t \in [0,T],$$

where $F^I(t)$ denotes the forest stock path obtained as a solution of system (28).

The following proposition shows the existence of positive coefficients of $F(t)$ and $D(t)$ in the transfer function.

Proposition 6 *There exist positive functions $v(t)$ and $w(t)$ in $[0,T]$ which induce the forestry country to choose in the short-run a deforestation rate leading to a sustainable forest path.*

Proof. Our aim is to prove the existence of functions $v(t)$ and $w(t)$, such that

$$v(t) \geq 0,\ \ w(t) \geq 0,\ \forall t \in [0,T],$$

and implying that the sustainable deforestation path $D^s(t)$ solves the following optimal control problem:

$$\max_{D(t)} \int_0^T e^{-\rho t}[R(D(t),F(t)) + v(t)F(t) - w(t)D(t)]\, dt + e^{-\rho T}\phi F(T),$$

$$\text{s.t.:} \quad \dot{F}(t) = -D(t) + rF(t),\ F(0) = F_0,$$

$$\int_0^T e^{-\rho t}[v(t)F(t) - w(t)D(t)]\, dt = J^T - J^s.$$

The isoperimetric constraint can be written equivalently as:

$$\dot{z}\,(t) = e^{-\rho t}[v(t)F(t) - w(t)D(t)]; \quad z\,(0) = 0; \quad z\,(T) = J^T - J^s, \tag{30}$$

where $\dot{z}\,(t)$ represents the transfer rate at t and $z\,(t)$ is the cumulative transfer from the start of the program till time t.

The current value Hamiltonian associated with the problem described above is

$$H^I = -a_1F^2 + a_2F - a_3D^2 + a_4D - a_5DF + a_6 + vF - wD + \lambda^I(-D + rF) + \mu(vF - wD),$$

where λ^I, μ denote the new costate variables associated with the forest stock and the transfer constraint (30), respectively.

Assuming interior solution, the necessary conditions of the Maximum Principle that guarantee the maximization of the new stream of revenues are given by:

$$\begin{aligned}
D^I &= \frac{1}{2a_3}\left(a_4 - a_5F^I - \lambda^I - w(1+\mu)\right), \\
\dot{F}^I &= -D^I + rF^I; \quad F^I\,(0) = F_0, \\
\dot{\lambda}^I &= a_5D^I + 2a_1F^I + (\rho - r)\,\lambda^I - v(1+\mu) - a_2; \quad \lambda^I\,(T) = \phi, \\
\dot{z} &= = e^{-\rho t}(vF^I - wD^I); \quad z(0) = 0, \quad z(T) = J^T - J^s, \\
\dot{\mu} &= 0.
\end{aligned} \tag{31}$$

From the last differential equation we have that the costate variable μ is constant along the time horizon. Let denote this constant by $\bar{\mu}$.

The incentive mechanism guaranteeing the implementation of the sustainable exploitation path is such that D^s and F^s, the deforestation and forest stock paths for the infinite horizon optimization problem, given in (10) and (9), satisfy the optimality conditions above.

From $D^I(t) = D^s(t) \forall t \in [0,T]$ we derive:

$$w(t)(1+\bar{\mu}) = \lambda^s(t) - \lambda^I(t) \quad \forall t \in [0,T]. \tag{32}$$

Subtracting the differential equations that describe the time evolution of the costate variables we have:

$$\dot{\lambda}^s(t) - \dot{\lambda}^I(t) = (\rho - r)(\lambda^s(t) - \lambda^I(t)) + v(t)(1+\bar{\mu}).$$

Differentiating with respect to time in (32) and replacing the expression of $\dot{\lambda}^s(t) - \dot{\lambda}^I(t)$, we have:

$$\dot{w}(t)(1+\bar{\mu}) = (\rho - r)w(t)(1+\bar{\mu}) + v(t)(1+\bar{\mu}).$$

Let assume that $1+\bar{\mu} \neq 0$, then the last differential equation reads[4]:

$$\dot{w}(t) = (\rho - r)w(t) + v(t). \tag{33}$$

Equation (33) gives $v(t) = \dot{w}(t) - (\rho - r)w(t)$. Substituting this expression in the differential equation (31), where F^I and D^I have been replaced by F^s and D^s,

[4]Note that if $1+\bar{\mu} = 0$, equation (32) implies:

$$\lambda^s(t) = \lambda^I(t) \quad \forall t \in [0,T].$$

Furthermore, the coefficient functions $v(t), w(t)$ of the transfer function do not appear in the optimality conditions. Therefore, the donor community cannot select these functions in order to induce the forestry country to follow a sustainable exploitation of the forest. The donor community attains its objective only if $\lambda^s(T) = \phi$.

respectively, we have:

$$\begin{aligned}
\dot{z}(t) &= e^{-\rho t}[(\dot{w}(t) - (\rho - r)w(t))F^s(t) - w(t)D^s(t)] \\
&= e^{-\rho t}\dot{w}(t)F^s(t) - e^{-\rho t}\rho F^s(t)w(t) + e^{-\rho t}rF^s(t)w(t) - e^{-\rho t}w(t)D^s(t) \\
&= e^{-\rho t}\dot{w}(t)F^s(t) - e^{-\rho t}\rho F^s(t)w(t) + e^{-\rho t}w(t)\dot{F}^s(t) \\
&= e^{-\rho t}\dot{w}(t)F^s(t) + w(t)\frac{d}{dt}(e^{-\rho t}F^s(t)) \\
&= \frac{d}{dt}[w(t)e^{-\rho t}F^s(t)].
\end{aligned}$$

Therefore,

$$z(t) = w(t)e^{-\rho t}F^s(t) + k_1,$$

where k_1 is a constant. The initial condition $z(0) = 0$ fixes the value of constant $k_1 = -w(0)F_0$, and then,

$$z(t) = w(t)e^{-\rho t}F^s(t) - w(0)F_0.$$

The final condition $z(T) = J^T - J^s$ leads to the following relationship:

$$w(T)e^{-\rho t}F^s(T) - w(0)F_0 = J^T - J^s. \tag{34}$$

From (32) we obtain:

$$w(T) = \frac{\lambda^s(T) - \lambda^I(T)}{1 + \bar{\mu}} = \frac{\lambda^s(T) - \phi}{1 + \bar{\mu}}. \tag{35}$$

Replacing this expression in (34) we derive:

$$w(0) = \frac{1}{F_0}\left(J^s - J^T + \frac{\lambda^s(T) - \phi}{1 + \bar{\mu}}e^{-\rho T}F^s(T)\right). \tag{36}$$

Therefore, we have to solve the differential equation (33) taking into account the initial and final conditions given in (36) and (35). Let us note that both conditions depend on the value of the parameter $\bar{\mu}$. To show this dependence let denote the initial and final conditions by $w_0(\bar{\mu}), w_T(\bar{\mu})$.

The solution of the differential equation (33) satisfying the initial condition is given by:

$$w(t) = w_0(\bar{\mu})e^{(\rho-r)t} + \int_0^t e^{(\rho-r)(t-s)}v(s)\,ds. \tag{37}$$

The final condition implies the following relationship:

$$w_T(\bar{\mu}) = w_0(\bar{\mu})e^{(\rho-r)T} + \int_0^T e^{(\rho-r)(T-s)}v(s)\,ds.$$

From this expression we get the value of parameter $\bar{\mu}$ which satisfies:

$$1 + \bar{\mu} = \frac{(\lambda^s(T) - \phi)(F_0 - F^s(T)e^{-rT})}{(J^s - J^T)e^{(\rho-r)T} + F_0 \int_0^T e^{(\rho-r)(T-s)}v(s)\,ds}, \tag{38}$$

where function $v(t)$ has to be selected such that the denominator of the expression above is not null.

From (37) it is straightforward to conclude that for any selection of $v(t)$ positive, if the initial condition $w_0(\bar{\mu})$ is non-negative, then $w(t) \geq 0 \forall t \in [0, T]$.

From (36) one has that $w_0(\bar{\mu})$ is non-negative if and only if

$$J^s - J^T + \frac{\lambda^s(T) - \phi}{1 + \bar{\mu}} e^{-\rho T} F^s(T) \geq 0. \tag{39}$$

Inequality (39) once the expression of $1 + \bar{\mu}$ given in (38) has been replaced establishes one constraint for the choice of function $v(t)$. This function has to be selected such that the following expressions have the same sign:

$$F_0 - F^s(T)e^{-rT}, \quad J^s - J^T + e^{-\rho T} F^s(T) \int_0^T e^{(\rho-r)(T-s)}v(s)\,ds.$$

The differential equation (26) can be rewritten as:

$$\dot{\lambda}^I(t) = (\rho - r)\lambda^I(t) + H(t, \bar{\mu}), \quad \lambda^I(T) = \phi,$$

where

$$H(t, \bar{\mu}) = 2a_1 F^s(t) - a_2 + a_5 D^s(t) - v(t)(1 + \bar{\mu}).$$

The solution of this initial value problem is given by:

$$\lambda^I(t) = \phi e^{-(\rho - r)(T-t)} - \int_t^T e^{-(\rho - r)(s-t)} H(s, \bar{\mu})\, ds.$$

■

The proposition shows that one can design a transfer function such that the forestry country would follow in the short term the sustainable deforestation policy. Note that we do not constrain the transfer $S(D, F)$ to be nonnegative at each instant of time. In the next example, $S(D, F)$ is positive for all t.

5. Numerical Illustration

To have a better insight into the results (deforestation strategies, transfers, etc.), we provide a numerical illustration. The value of the parameters, which are mostly inspired from Van Soest and Lensink (2000), are the following:

$$\bar{P} = 45000,\ \bar{P}_A = 150,\ \bar{Z} = 60,\ F_0 = 2000,\ \theta = 20,\ \gamma = 0.15,$$
$$\alpha = 0.1,\ \beta = 0.1,\ \rho = 0.1,\ r = 0.2,\ \phi = 17000, T = 5.$$

The steady-state values for deforestation, forest stock and shadow price of the forest are:

$$D^* = 343.42, \qquad F^* = 1717.10, \quad \text{and} \quad \lambda^* = 25204, .$$

The time paths of the forest stock, the deforestation rate and the shadow price in the different scenarios are given by:

$$\begin{aligned}
F^s(t) &= F^I(t) = F^* + 282.89e^{-0.93t}, \\
F^T(t) &= F^* + 286.84e^{-0.93t} - 3.94e^{1.03t}, \\
D^s(t) &= D^I(t) = D^* + 318.53e^{-0.93t}, \\
D^T(t) &= D^* - 322.97e^{-0.93t} + 3.26e^{1.03t}, \\
\lambda^s(t) &= \lambda^* - 18682e^{-0.93t}, \\
\lambda^T(t) &= \lambda^* - 18942e^{-0.93t} - 47.45e^{1.03t}, \\
\lambda^I(t) &= [24.431\left(1 - e^{-0.1(5-t)}\right) + 18.7e^{-0.1t}\left(0.02 - e^{-0.83t)}\right)] \times 10^3
\end{aligned}$$

Figures (1) to (3) show these time paths. In all the figures continuous line corresponds to the sustainable scenario, while the dash line denotes the finite horizon scenario. These figures allow the following comments:

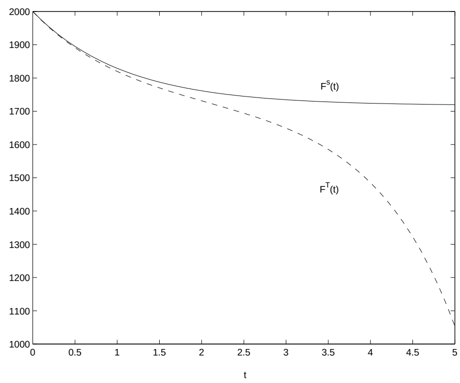

Figure 1: Forest stock time paths

- In the sustainable scenario, the forest stock and the deforestation rate time paths decrease overtime towards their steady-state values (see figures (1) and (2)). In the finite horizon scenario, the forestry country accelerates deforestation as the end of horizon approaches. Consequently, forest stock decreases sharply overtime. This result is expected in a finite-horizon optimization problem and justifies the idea of implementing an incentive policy to slow down deforestation.

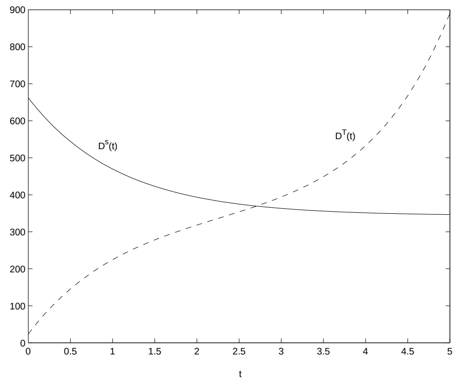

Figure 2: Deforestation time paths

- Figure (3) indicates that the shadow price of the forest stock is always increasing in both the sustainable and incentive scenarios. This is intuitive; as the forest stock decreases over time, the value of an additional unit of forest increases. Although the forest stock is also decreasing in the finite horizon scenario, its shadow price is not monotone. It is increasing in the beginning and decreasing on a final time interval. This last portion can be, again, explained by an end of horizon argument.

The revenues paths are as follows:

$$
\begin{aligned}
R^s(t) &= [-5.15e^{-1.85t} + 7.32e^{-0.93t} + 22.62] \times 10^6. \\
R^T(t) &= -29.93e^{0.1t} - 207.42e^{2.05t} + 91997e^{1.03t} + \\
&\quad [-5.30e^{-1.85t} + 7.42e^{-0.93t} + 22.62] \times 10^6, \\
R^I(t) &= R^s(t) + S(D^s(t), F^s(t)).
\end{aligned}
$$

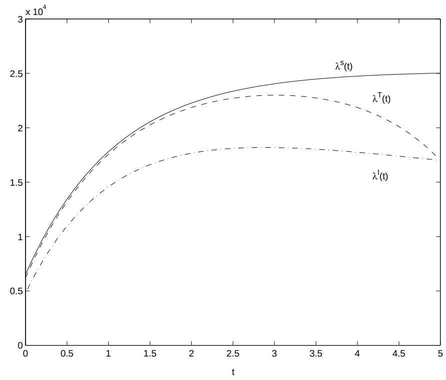

Figure 3: Shadow price time paths

It is easy to see that $R^T(t) > R^s(t), \forall t \in [0,T]$. The total loss for the *South* if it applies the sustainable deforestation policy over the time interval is given by $J^T - J^s = 1.40 \times 10^6$. This total amount has to be compensated for by the *North* by a stream of transfers given by

$$S(D,F) = v\,(t)\,F(t) - w\,(t)\,D\,(t)\,.$$

The last proposition showed that the above function can be constructed by first fixing $v(t)$ and next determining $w(t)$. To illustrate, let us consider a constant specification for function $v(t) = \bar{v} = 500$. It is easy to check that $w(t)$ would be given by

$$w(t) = -4522.8e^{-0.1t} + 5000,$$

and that it is positive for all t. The time path for the transfer function reads

$$S(t) = [-8.58 - 14.5e^{-0.93t} + 15.5e^{-0.1t} + 14.4e^{-1.03t}] \times 10^5.$$

The evolution of the transfers overtime is given by

$$\dot{S}(t) = [13.49e^{-0.93t} - 1.55e^{-0.1t} - 14.83e^{-1.03t}] \times 10^5,$$

which is negative. Thus the incentive policy in this example is to start supporting the *South* at a high level and decreasing it overtime (see figure (4)).

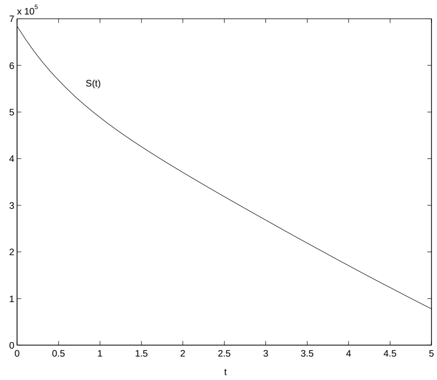

Figure 4: Transfers time path

6. Conclusion

Using a detailed model showing the trade-off between the agriculture and forestry activities (deforestation and agriculture productivity), we showed that the international community can encourage the forestry countries to participate in a program aiming at a better forest conservation while compensating it for the revenue loss it can bear from doing so. We also showed that by using incentive transfer mechanisms, the donor community or *North* can enforce a sustainable forest exploitation in the short-run.

For further research, we can consider efforts to regenerate the forest. This would require a model incorporating a lag structure to handle correctly the link between replanting trees and later production of the resource. The *North* could also contribute financially to such a program in a principal-agent setting.

Appendix

- **Constants** $A_i, i = 1, 2, 3$ **in Proposition 1**.

$$A_1 = -\frac{(F_0 - F^*)^2 \left(4a_1 + (2a_5 + a_3(2r + \Delta - \rho))(2r + \Delta - \rho)\right)}{4},$$

$$A_2 = \frac{(F_0 - F^*)\left(F^*(a_5^2 - 4a_1 a_3) + \lambda^*(a_5 + a_3(2r + \Delta - \rho)) + 2a_2 a_3 - a_4 a_5\right)}{2a_3},$$

$$A_3 = \frac{(a_4 - a_5 F^* - \lambda^*)(a_4 - a_5 F^* + \lambda^*) + 4a_3\left(-a_1 (F^*)^2 + a_2 F^* + a_6\right)}{4a_3}.$$

- **Constants** $C_i, i = 1, 2$ **and** $B_j, j = 1, \ldots, 6$ **in Proposition 2**.

$$C_1 = \frac{(\phi - \lambda^*)((2r - \rho + \Delta)a_3 + a_5) - (a_5^2 - 4a_1 a_3)(F^* - F_0)\, e^{(\Delta + \rho)\frac{T}{2}}}{(a_3(\Delta + 2r - \rho) + a_5)\, e^{(\rho - \Delta)\frac{T}{2}} + (a_3(\Delta - 2r + \rho) - a_5)\, e^{(\Delta + \rho)\frac{T}{2}}},$$

$$C_2 = \frac{(a_3(\Delta - 2r + \rho) - a_5)(\phi - \lambda^*) + (a_5^2 - 4a_1 a_3)(F^* - F_0)\, e^{(\rho - \Delta)\frac{T}{2}}}{(a_3(\Delta + 2r - \rho) + a_5)\, e^{(\rho - \Delta)\frac{T}{2}} + (a_3(\Delta - 2r + \rho) - a_5)\, e^{(\Delta + \rho)\frac{T}{2}}},$$

$$B_1 = -\frac{C_2\left(a_3 a_5^2 \Delta^2 C_2 + 2C_1(a_5^2 - 4a_1 a_3)\left(-4a_1 + 2a_5(-2r + \rho) + a_3\left(\Delta^2 - (2r - \rho)^2\right)\right)\right)}{4(a_5^2 - 4a_1 a_3)^2},$$

$$B_2 = \frac{C_2^2\left(a_3 a_5^2 \Delta^2 + (a_5^2 - 4a_1 a_3)(2(a_5 + 2a_1) + a_3(2r + \Delta - \rho))(2r + \Delta - \rho)\right)}{4(a_5^2 - 4a_1 a_3)^2},$$

$$B_3 = -\frac{C_2\left((a_5^2 - 4a_1 a_3)\lambda^* + (2a_2 a_3 - a_4 a_5 + (a_5^2 - 4a_1 a_3)F^*)(a_5 + a_3(\Delta + 2r - \rho))\right)}{2a_3(a_5^2 - 4a_1 a_3)},$$

$$B_4 = \frac{C_1^2\left(4a_1 + (2a_5 + a_3(2r - \Delta - \rho))(2r - \Delta - \rho)\right)}{4(a_5^2 - 4a_1 a_3)},$$

$$B_5 = -\frac{C_1\left((a_5^2 - 4a_1 a_3)\lambda^* + (2a_2 a_3 - a_4 a_5 + (a_5^2 - 4a_1 a_3)F^*)(a_5 + a_3(2r - \rho - \Delta))\right)}{2a_3(a_5^2 - 4a_1 a_3)},$$

$$B_6 = A_3.$$

- **Constants** $M_i, i = 1, 2$ **in Proposition 3**.

$$M_1 = \frac{C_1^2\left(4a_1 + \left(2a_5 + a_3\left(2r - \Delta - \rho\right)\right)\left(2r - \Delta - \rho\right)\right)}{4\left(a_5^2 - 4a_1a_3\right)} + \frac{\left(F_0 - F^*\right)^2\left(4a_1 + \left(2a_5 + a_3\left(2r + \Delta - \rho\right)\right)\left(2r + \Delta - \rho\right)\right)}{4},$$

$$M_2 = -\frac{C_1\left(\left(a_5^2 - 4a_1a_3\right)\lambda^* + \left(2a_2a_3 - a_4a_5 - \left(4a_1a_3 - a_5^2\right)F^*\right)\left(a_5 + a_3\left(2r - \rho - \Delta\right)\right)\right)}{2a_3\left(a_5^2 - 4a_1a_3\right)} - \frac{\left(F_0 - F^*\right)\left(F^*\left(a_5^2 - 4a_1a_3\right) + \lambda^*\left(a_5 + a_3\left(2r + \Delta - \rho\right)\right) + 2a_2a_3 - a_4a_5\right)}{2a_3}.$$

ESSAY 3

Characteristic Functions, Coalitions Stability and Free-riding in a Game of Pollution Control

1. Introduction

The call for international cooperation is motivated mainly by environmental and economic efficiency to deal with global warming. The environmental efficiency is related to the number of countries that are committed to reduce their emission levels. The larger this number is, the more important will the greenhouse abatement be and the easier will the stabilization of the concentration level of these gases be. On the other hand, the economic efficiency is a result of the involvement of a very large geographical area in the emission abatement agreement, which enables us to apply the measures of reduction where it costs less. Consequently, the larger will the cooperation be between different nations, the lower will be the cost of the greenhouse abatement.

In reality, however, at an individual level, the countries may have different incentives from the global ones. In fact, even if a large cooperation can be more efficient, any country can be tempted to benefit from the environmental improvement accomplished by the existing cooperation without bearing any cost itself. This is one of the most common problems that can block international cooperation, which has been addressed in the game theory literature as the Free-riding problem.

For instance, in the game theory literature, the design of an International Environmental Agreement (IEA), that is a mechanism allocating to each country a collectively suitable emissions policy supported possibly by a monetary transfer, has been addressed following two lines of thought.

The first one adopts cooperative game theory as the analytical framework. The allocation problem is solved following a two-step methodology. First, one computes the Pareto-optimal emission levels and second, one uses a solution concept based on cooperative game theory (e.g., Shapley value, core, etc.) to allocate to each player his share of the total optimized cooperative payoff. The remaining issue, is to find the right allocation function that guarantees the stability of the formed solution in the core sense. (See for instance Bahn et al. (1998), Chander and Tulkens (1992), Currarini and Tulkens (1998), Eyckmans and Tulkens (1999), Filar and Gaertner (1997), Gaertner (2001), Germain et al. (1998), Jørgensen and Zaccour (2000, 2001), Kaitala et al. (1995) and Petrosjan and Zaccour (2000, 2003)). In this approach, the stability of the coalition is passive in the sense that the number of participating countries is exogenous. In other words, this approach supposes the existence of a large number of countries that are predisposed to sign the agreement, of which the nomination of grand coalition approach.

A second line of research, exemplified by, e.g., Barrett (1990, 1992, 1994-b) and Carraro and Siniscalco (1993), sees the problem of designing (or signing) an IEA from the perspective of coalitions stability, a concept due to d'Aspremont and Gabzewicz (1986) which has its root in the cartel problem in industrial organization literature. The stability of an IEA is ensured by two tests: the first one intends to see if it is in the interest of an already formed group of signatories to enlarge the IEA to new members (entry test); the second intends to check if it is in the interest of a player to remain in the coalition (exit test). The general message carried out in this literature is that only a small number of countries will end up signing an IEA (irrespective of the total number of countries), i.e., only a small stable coalition can emerge. It is worth noticing that this literature shows also that the introduction of monetary transfers does not help in enlarging the number of signatories of an IEA. This approach is also known as the small coalition approach.

In the environmental games, the debate about the possibility to deter free-riding and guarantee the stability of a large coalition is not closed yet. Some of the related work in this field are those by Barret (1990, 1992, 1994-b), Chander and Tulkens (1995, 1997), Black et al. (1993), Carraro and siniscalco (1993), Eykmans (2001), Helm (2001).

In a recent essay, Tulkens (1998) discussed and contrasted the premises on which the two approaches rest. He also raised the question if it is ever possible to reconcile them. He concluded his essay on a positive note saying that the characteristic function, a corner stone of cooperative game theory, may be the tool that will permit to achieve this.

This essay does two things: first, it discusses different definitions of the characteristic function of a cooperative game for the allocation of countries' welfare shares when signing an IEA; and second, it addresses the issue of reconciliation raised by Tulkens (1998). These issues are important from both modeling and empirical points of view. Indeed, there is no unique way to define the characteristic function of a cooperative game and therefore it is of interest to discuss the different options available and their impact on the final allocation result. Further, if a cooperative game approach is intended to prescribe environmental policies and cost sharing mechanisms, then it has to be credible in the sense that these policies and mechanisms need to satisfy a certain number of properties, among them deterring free-riding. Our results show that irrespective of the definition adopted for the characteristic function of the cooperative game, one cannot deter free-riding in the sense of the stable coalition concept. As a consequence, we argue that it is not at reach to reconcile the two approaches (cooperative games and stable coalitions).

The rest of the essay is organized as follows: Section 2 describes the players, their strategies and payoffs. Section 3 discusses three ways of defining the characteristic function of a cooperative game aiming at an IEA and states some results. Section 4 deals with the issue of coalitions stability. Section 5 concludes.

2. Players, strategies and payoffs

To keep the emphasis on issues related to the definition of the characteristic function, we consider a simple economics/environment interactions model which nevertheless captures one of the main ingredients of the international debate on pollution, namely that emission by one country damages the environment of all players.

Let $N = \{1, \cdots n\}$ be the set of players (countries, regions, etc.) involved in a negotiation aiming at an IEA on pollution control. We assume that the production of goods by player i depends on only one variable input, namely pollution that we denote by e_i. Therefore, we can express the net revenue from production (revenue minus production cost) as a function of emissions only. Let this function be denoted by $R_i(e_i)$. Assume also that emissions by any country damage the common environment. Each player then incurs a damage cost denoted $D_i(\sum_{i=1}^n e_i)$. We assume that $R_i(e_i)$ is concave increasing and $D_i(\sum_{i=1}^n e_i)$ is convex increasing. The payoff of player i depends on all players' emissions and is given by

$$W_i(e_1, \cdots, e_n) = R_i(e_i) - D_i(\sum_{i=1}^{n} e_i). \tag{1}$$

For computation purposes, we make the further simplifying assumption that revenue and damage cost functions are as follows

$$R_i(e_i) = b_i e_i - 1/2 e_i^2$$
$$D_i(\sum_{i=1}^{n} e_i) = d_i \sum_{i=1}^{n} e_i$$

where b_i and d_i are positive constants. Under this specification, the objective of player i becomes

$$W_i(e_1, \cdots e_n) = b_i e_i - 1/2 e_i^2 - d_i \sum_{i=1}^{n} e_i. \tag{2}$$

The strategy space of player i is defined as $[0, b_i]$. The interpretation of the lower bound is obvious and the upper bound corresponds to the optimal pollution level when player i maximizes his payoff without taking into account his damage cost.

3. Characteristic functions

Let $\Gamma(N, v, X)$ be a cooperative game where $N = \{1, \cdots n\}$ is the set of players involved in the negotiation of an IEA, $S \subseteq N$ is a coalition, $v(S)$ is the *Characteristic Function* (CF), and X is the set of *imputations*.

The characteristic function measures the payoff (or strength) of a coalition. Obviously in the context of pollution control, a game with externalities, this outcome depends on the actions taken by the players who are not members of this coalition. We shall assume three different types of behavior of left-out-players (LOP) later on. To be attractive, cooperation between individual players or coalitions of players must create an added value to them. This can be translated by one of the following properties.

Definition 1 *The function $v(S)$ is superadditive, if*

$$v(S \cup T) \geq v(S) + v(T), \qquad \forall S, T \subseteq N, \quad S \cap T = \emptyset, \tag{3}$$

where $v(\varnothing) = 0$.

Superadditivity simply means that when two coalitions join forces, they can achieve at least the same payoff than by acting separately. Note that (3) must be satisfied for all possible coalitions. The following definition provides a weaker condition which states that the worth of the grand coalition must be at least as large as the worth of the members of any partition of the set N.

Definition 2 *The function $v(S)$ is cohesive, if*

$$v(N) \geq \sum_{m=1}^{M} v(T_m), \quad \textit{for every partition } \{T_1, \ldots T_M\} \textit{ of } N, \tag{4}$$

where $v(\varnothing) = 0$.

The definition of a cooperative game requires that function $v(S)$ satisfies (3) or (4) but does not specify how to compute $v(S)$. A large part of the essay is precisely devoted to this issue. Before, we define the last ingredient of a cooperative game.

Definition 3 *A vector $x = (x_1, ..., x_n)$ is an imputation if it satisfies*

$$x_i \geq v(\{i\}), \forall i \in N, \tag{5}$$

$$\sum_{i=1}^{n} x_i = v(N). \tag{6}$$

An imputation is a vector of players' outcomes. Its definition refers to individual and group rationality. Individual rationality means that a player will not accept an outcome which is not at least equal to what he can get by acting alone as measured by his characteristic function value. Group rationality states that the total cooperative gain, when the grand coalition forms, is shared. From a negotiation perspective, the set of imputations reduces the number of potentially acceptable agreements. This set is seldom a singleton and therefore one needs other properties to predict the final issue of the game. This is precisely the objective pursued by the different solution concepts of a cooperative game. For instance, the *core* selects those imputations which are undominated and the *Shapley value* selects one imputation satisfying certain axioms, among them fairness. The core may be empty or may contain many imputations. It is well known that for an imputation to be in the core it must satisfy

$$x_i \geq v(\{i\}), \forall i \in N, \ and \tag{7}$$

$$\sum_{i \in S} x_i \geq v(S), \forall S \subseteq N. \tag{8}$$

We now move to the issue of computation of the characteristic function values. The CF aims to assess the strength of coalitions and this strength (strategic force, payoff, etc.) depends, among other things, on the behavior of the left-out-players (LOP). We shall investigate three different assumptions regarding the way LOP react to the formation of a coalition that excludes them.

In the first case, we assume that coalition S plays a noncooperative game against LOP acting individually. If one denotes by s the number of players belonging to S, then one has a game with $n - s + 1$ players. The payoffs of coalition S and LOP correspond to Nash equilibrium outcomes. We shall refer to this case by *PNE* for Partial Nash Equilibrium.

In the second case we assume that LOP players form also a coalition and payoffs are then obtained as Nash equilibrium outcomes of a noncooperative game between S

and $N\backslash S$. The computation of each characteristic function value involves therefore two players. We shall refer to this case by *NE* for Nash Equilibrium.

Finally, we consider the case where left-out-players form a coalition which maximizes their payoff without taking into account their damage cost. Ignoring damage cost will lead obviously to higher emissions by LOP. We interpret this behavior by LOP as a way of punishing players belonging to S for not including them in the coalition. We shall refer to this case by *PB* (for Punishing Behavior).

We use the following notation in the sequel:

$W^x(S)$: maximized payoff of coalition S for $x = PNE, NE, PB$.

$W_i^x(S)$: maximized payoff of $i \in S$ for $x = PNE, NE, PB$.

$Y_j^x(S)$: maximized payoff of player $j \notin S$ for $x = PNE, NE, PB$.

$Y^x(S) = \sum_{j \notin S} Y_j^x(S)$.

$v^x(S)$: characteristic function value for S for $x = PNE, NE, PB$.

$D_Z = \sum_{i \in Z} d_i, Z \subseteq N$.

$B_Z = \sum_{i \in Z} b_i, Z \subseteq N$.

Although the assessment of the strength of any coalition $S \subset N$ may vary with x, the payoff of the grand coalition N must be the same for all x (the set of LOP being void in all cases). We assume that the objective of the grand coalition is to maximize the sum of welfare of all players. This optimization provides the sought vector of optimal emission levels, that is the levels of an IEA, and the total collective payoff to be shared. The following Lemma gives the result.

Lemma 4 *Assuming interior solutions, the international optimal emission levels are given by*

$$e_i = b_i - D_N, \quad \forall i \in N, \tag{9}$$

and the optimal total payoff is given by

$$W^x(N) = \frac{1}{2}\sum_{i\in N} b_i^2 + \frac{n}{2}D_N^2 - D_N B_N, \quad \forall x. \tag{10}$$

Proof. The strictly concave optimization problem of the grand coalition is given by

$$\max \sum_{i\in N} W_i(e_1, \cdots, e_n) = \max \sum_{i=1}^{n}(b_i e_i - \frac{1}{2}e_i^2) - \sum_{i\in N} d_i \sum_{i\in N} e_i. \tag{11}$$

It suffices to differentiate with respect to e_i to get (9) as unique solution and to substitute in (11) to get (10). ■

As it is seen from (9), the optimal international emission strategy prescribes to each player to set his emission level taking into account the sum of all players marginal damage costs.

3.1 Partial Nash Equilibrium

Recall that the assumption here is that when coalition S forms, the left-out-players play individually. The payoffs are then given as Nash equilibrium outcomes in an $n - s + 1$ player-game. In particular, note that the payoffs $W_i^{PNE}(\{i\}), i = 1, \cdots, n$, are the Nash equilibrium outcomes of a fully n-player noncooperative game. The following proposition summarizes the results.

Proposition 5 *Characterization of Partial Nash Equilibrium assuming interior solutions*

(i) Equilibrium emission strategies are given by

$$e_i^{PNE} = b_i - D_S, \quad \forall i \in S, \tag{12}$$

$$e_j^{PNE} = b_j - d_j, \quad \forall j \in N\backslash S. \tag{13}$$

(ii) Equilibrium outcomes are given by

$$W^{PNE}(S) = \frac{1}{2}\sum_{i\in S} b_i^2 + \frac{s}{2}D_S^2 + D_S\left(D_{N\backslash S} - B_N\right), \tag{14}$$

$$Y_j^{PNE}(S) = \frac{1}{2}(b_j^2 - d_j^2) + d_j\left(sD_S + D_{N\backslash S} - B_N\right), \forall j \in N\backslash S. \tag{15}$$

Proof. The optimization problems of S and $j \in N\backslash S$ are as follows:

$$\max \sum_{i\in S}(b_i e_i - \frac{1}{2}e_i^2) - \sum_{i\in S} d_i \sum_{l\in N} e_l, \tag{16}$$

$$\max(b_j e_j - \frac{1}{2}e_j^2) - d_j \sum_{l\in N} e_l, \quad j \in N\backslash S. \tag{17}$$

Differentiating with respect to emissions and equating to zero leads to the result in (i). Substituting for equilibrium emissions in the objectives of the optimization problems leads to the result in (ii). ■

This proposition shows that when a coalition forms, each member sets his emission strategy taking into account the sum of marginal damage costs (D_S) of all coalition's members. This result carries the same message as the one advocated in the fully cooperative case but at the level of coalition S. Note that (14) is the total outcome of coalition S. Its division among his members is not an issue for the moment. The individual payoff of players outside the coalition can be interpreted as the outcome obtained by a player when he free rides i.e., when he decides to stay out of the coalition. Of interest is this outcome when computed for player i when $S = N\backslash\{i\}$. Indeed it assesses the outcome of that player when he is the only one to free ride or to play noncooperatively.

Although this computation permits to evaluate the strength of all possible coalitions under this scenario, it remains to be checked if this leads to a well defined characteristic function.

Proposition 6 *The function* $v^{PNE}(S) = W^{PNE}(S)$ *is superadditive.*

Proof. We need to show that $v^{PNE}(S)$ satisfies (3). (Note that obviously $v^{PNE}(\emptyset) = 0$). Let s and t denote the number of players in respectively coalition S and T. To prove that $v^{PNE}(S)$ is superadditive, use (14) to compute for $\forall S, T \subseteq N$, $S \cap T = \emptyset$

$$v^{PNE}(S \cup T) - v^{PNE}(S) - v^{PNE}(T) = \frac{1}{2} \sum_{i \in S \cup T} b_i^2 + \frac{(s+t)}{2} D_{S \cup T}^2$$
$$-D_{S \cup T} B_N + D_{S \cup T} D_{N \setminus S \cup T} - \left(\frac{1}{2} \sum_{i \in S} b_i^2 + \frac{s}{2} D_S^2 - D_S B_N + D_S D_{N \setminus S} \right)$$
$$- \left(\frac{1}{2} \sum_{i \in T} b_i^2 + \frac{t}{2} D_T^2 - D_T B_N + D_T D_{N \setminus T} \right),$$

which after lengthy but straightforward computations reduces to

$$v^{PNE}(S \cup T) - v^{PNE}(S) - v^{PNE}(T) = (s+t-2) D_S D_T + \frac{t}{2} D_S^2 + \frac{s}{2} D_T^2,$$

which is obviously positive. ■

Given the values of this characteristic function, one can use any solution concept of cooperative games to allocate the grand coalition payoff among the players. In the environmental context (see, e.g., Germain et al. (1998) and Petrosjan and Zaccour (2003)) a particular attention has been devoted to imputations in the core since such outcomes are stable in the sense that there is no coalition that can block them. The core may however be empty. Schematically this occurs when intermediate coalitions are "too strong", i.e., enlarging to the grand coalition does not create sufficient incremental wealth to satisfy all players' appetite. Here the core is not void. A sufficient condition (Shapley (1972)) for nonemptiness is that the game be convex, that is

$$v(S \cup T) + v(S \cap T) \geq v(S) + v(T), \quad \forall S, T \subseteq N.$$

Proposition 7 *The cooperative game with characteristic function $v^{PNE}(S)$ is convex.*

Proof. For $\forall S, T \subseteq N$, lengthy by again straightforward algebraic calculations give

$$\begin{aligned} & v^{PNE}(S \cup T) + v^{PNE}(S \cap T) - v^{PNE}(S) - v^{PNE}(T) \\ = & \frac{t}{2}\left(D_S^2 - D_{S\cap T}^2\right) + \frac{s}{2}\left(D_T^2 - D_{S\cap T}^2\right) \\ & +(s+t-2) D_{S\backslash S\cap T} D_{T\backslash S\cap T} + \frac{|S\cap T| + 4}{2} D_{S\cap T}^2, \end{aligned}$$

which is obviously positive. ■

To conclude on this scenario, we have established the (expected) result that a coalition takes into account the sum of damage costs of its members in its emissions strategy, that a suitable characteristic function is at hand and that the core is non empty.

3.2 Nash Equilibrium

Under this scenario, the assessment of the strength of a coalition S is obtained as its Nash equilibrium payoff in a noncooperative game between this coalition and its complement $(N\backslash S)$. The following proposition summarizes our result.

Proposition 8 *Characterization of Nash Equilibrium assuming interior solutions:*

(i) Equilibrium emission strategies are given by

$$e_i^{NE} = b_i - D_S, \quad \forall i \in S, \tag{18}$$

$$e_j^{NE} = b_j - D_{N\backslash S}, \quad \forall j \in N\backslash S. \tag{19}$$

(ii) Equilibrium outcomes are given by

$$W^{NE}(S) = \frac{1}{2}\sum_{i\in S} b_i^2 + \frac{s}{2}D_S^2 + D_S\left((n-s)D_{N\backslash S} - B_N\right), \quad (20)$$

$$Y^{NE}(S) = \frac{1}{2}\sum_{i\in N\backslash S} b_i^2 + \frac{(n-s)}{2}D_{N\backslash S}^2 + D_{N\backslash S}\left(sD_S - B_N\right). \quad (21)$$

Proof. The optimization problems of S and $j \in N\backslash S$ are as follows:

$$\max \sum_{i\in S}(b_i e_i - \frac{1}{2}e_i^2) - \sum_{i\in S} d_i \sum_{l\in N} e_l; \quad (22)$$

$$\max \sum_{j\in N\backslash S}(b_j e_j - \frac{1}{2}e_j^2) - \sum_{j\in N\backslash S} d_j \sum_{l\in N} e_l\ . \quad (23)$$

Differentiating with respect to emissions and equating to zero leads to the result in (i). Substituting for equilibrium emissions in the objectives of the optimization problems leads to the result in (ii). ∎

The proposition establishes that emissions by any player depend on the sum of marginal damage costs of all players belonging to the same coalition. The following proposition shows that the payoff function is not cohesive and therefore not superadditive.

Proposition 9 *The function $W^{NE}(S)$ is not cohesive.*

Proof. To show this, it suffices to provide a counter-example. Let the n players be identical, that is $d_i = d$ and $b_i = b, \forall i \in N$. Under this assumption (20) becomes

$$W^{NE}(S) = \frac{s}{2}\left(b^2 + s^2d^2 - 2ndb + 2(n-s)^2d^2\right). \quad (24)$$

Consider the partition of the set of players into n singletons. Now use (24) to compute $W^{NE}(\{i\})$, that is the payoff of a coalition of one player

$$W^{NE}(\{i\}) = \frac{1}{2}\left(b^2 + d^2 - 2ndb + 2(n-1)^2d^2\right). \quad (25)$$

Compute

$$W^{NE}(N)-\sum_{i=1}^{n}W^{NE}(\{i\})=\frac{n}{2}\left(b^2+n^2d^2-2ndb\right)$$
$$-\frac{n}{2}\left(b^2+d^2-2ndb+2(n-1)^2d^2\right)=\frac{nd^2}{2}\left(-n^2-3+4n\right),$$

which is obviously negative for $n>3$. Hence the result. ■

Given that the computation of each value of $W^{NE}(S)$ involves a game between two coalitions, we shall further specialize the notion of cohesiveness and introduce the following

Definition 10 *The function $W(N)$ is weakly cohesive if*

$$W(N)\geq W(S)+W(T),\quad \forall S,T\subseteq N, S\cup T=N, S\cap T=\emptyset$$

As it is readily seen from the definition, the payoff of the grand coalition is at least as large as the sum of payoffs of the members of any two complement subsets of N. We have the following result.

Proposition 11 *The function $W^{NE}(S)$ is weakly cohesive.*

Proof. Compute for $\forall S,T\subseteq N$ such that $S\cup T=N$ and $S\cap T=\emptyset$

$$W^{PNE}(N)-W^{PNE}(S)-W^{PNE}(T)=\frac{n}{2}D_N^2-\frac{s}{2}D_S^2$$
$$+(n-s)\,D_SD_T-\frac{t}{2}D_T^2-(n-t)\,D_SD_T$$
$$=\frac{t}{2}D_S^2+\frac{s}{2}D_T^2+(s+t)\,D_SD_T,$$

which is obviously positive and hence the result. ■

To conclude on this scenario, we have the result that every player takes into account the sum of damage costs of all members belonging to the same coalition.

Note however that the payoff function $W^{PNE}(S)$ is not cohesive and therefore we cannot claim that a proper cooperative game can be defined.

3.3 Punishing Behavior

The assumption here is that LOP form an anti-coalition and agree on setting their emissions levels at their upper bounds. Recall that the latter have been defined as those resulting from the maximization of revenue only (i.e., disregarding damage cost). This behavior is to be interpreted as a punishment by LOP of coalition S for not including them in the coalition. There is an obvious link between this approach and the way the characteristic function is computed by von Neumann and Morgenstern (1944) in their seminal book. Indeed, von Neumann and Morgenstern assumed that left-out-players form an anti-coalition with the sole objective of minimizing the payoff of coalition S. This results in a characteristic function value $v(S)$ providing the minimum guaranteed payoff for that coalition. We retain here the idea of an antagonistic anti-coalition but at what seems to be a credible level. The strategies and payoffs are given in the following proposition.

Proposition 12 *Characterization of Punishing Behavior solution, Assuming interior solutions:*

(i) Equilibrium emission strategies are given by

$$e_i^{PB} = b_i - D_S, \quad \forall i \in S, \tag{26}$$

$$e_j^{PB} = b_j, \quad \forall j \in N \backslash S. \tag{27}$$

(ii) Equilibrium outcomes are given by

$$W^{PB}(S) = \frac{1}{2}\sum_{i\in S} b_i^2 + \frac{s}{2}D_S^2 - D_S B_N \ , \tag{28}$$

$$Y^{PB}(S) = \frac{1}{2}\sum_{j\in N\backslash S} b_j^2 + D_{N\backslash S}\left(sD_S - B_N\right). \tag{29}$$

Proof. Obviously every left-out-player sets his emission according to (27). The optimization problem coalition S hence becomes

$$\max \sum_{i\in S}(b_i e_i - \frac{1}{2}e_i^2) - \sum_{i\in S} d_i \left(\sum_{i\in S} e_i + \sum_{j\in N\backslash S} b_j\right).$$

Differentiating with respect to e_i and equating to zero leads to (26). Straightforward computation of the payoffs of S and $N\backslash S$ leads to the result in (ii). ■

The following two propositions show respectively that the function $v^{PB}(S) = W^{PB}(S)$ is superadditive and convex. Note that obviously $v^{PB}(\emptyset) = 0$.

Proposition 13 *The function $v^{PB}(S) = W^{PB}(S)$ is superadditive.*

Proof. We need to show that $v^{PB}(S)$ satisfies (3). To prove that this function is superadditive, use (28) to compute for $\forall S, T \subseteq N$, $S \cap T = \emptyset$

$$v^{PB}(S\cup T) - v^{PB}(S) - v^{PB}(T) = \frac{1}{2}\sum_{i\in S\cup T} b_i^2 + \frac{(s+t)}{2}D_{S\cup T}^2 - D_{S\cup T}B_N$$
$$- \left(\frac{1}{2}\sum_{i\in S} b_i^2 + \frac{s}{2}D_S^2 - D_S B_N\right) - \left(\frac{1}{2}\sum_{i\in T} b_i^2 + \frac{t}{2}D_T^2 - D_T B_N\right).$$

Straightforward manipulations leads to

$$v^{PB}(S\cup T) - v^{PB}(S) - v^{PB}(T) = \frac{t}{2}D_S^2 + \frac{s}{2}D_T^2 + (s+t)D_S D_T > 0$$

. ■

Proposition 14 *The cooperative game with characteristic function $v^{PB}(S)$ is convex.*

Proof. Lengthy but still straightforward computations give for all coalitions $S, T \subseteq N$

$$v^{PB}(S \cup T) + v^{PB}(S \cap T) - v^{PB}(S) - v^{PB}(T) = \frac{t}{2}\left(D_S^2 - D_{S\cap T}^2\right)$$
$$+\frac{s}{2}\left(D_T^2 - D_{S\cap T}^2\right) + \frac{|S\cap T|}{2}D_{S\cap T}^2 + (s+t)D_{S\setminus S\cap T}D_{TS\setminus S\cap T} > 0.$$

■

To conclude on this scenario, we have established that a coalition takes into account the sum of damage costs of its members in its emissions strategy and that a convex characteristic function implying a non empty core is at hand.

3.4 Some Comparisons

We wish here to make some comparisons between emissions levels as well as between payoffs generated under the three scenarios considered.

Proposition 15 *Comparisons of total emissions and outcomes*

(i) *Total emissions levels $TE^x, x = PNE, NE, PB$ for a given coalition S are related as follows:*

$$TE^{NE}(S) < TE^{PNE}(S) < TE^{PB}(S).$$

(ii) *Outcomes are related as follows:*

$$W^{NE}(S) > W^{PNE}(S) > W^{PB}(S).$$

Proof. From (12)-(13)-(18)-(19) and (26)-(27) one computes the total emissions

$$\begin{aligned} TE^{PNE}(S) &= B_N - sD_S - D_{N\setminus S}, \\ TE^{NE}(S) &= B_N - sD_S - (n-s)D_{N\setminus S}, \\ TE^{PB}(S) &= B_N - sD_S, \end{aligned}$$

and hence the result in (i). Straightforward comparisons of (14), (20) and (28) lead to (ii). ■

This proposition shows that from environmental and outcome perspectives, the best situation, assuming that only partial cooperation will take place, is when players behave à la *NE*. Of course, when one adopts a cooperative game approach, the expectation is that the grand coalition will indeed form.

Proposition 16 *The core of the cooperative game* $\Gamma(N, v^{PNE}, X)$ *is a subset of the core of the cooperative game* $\Gamma(N, v^{PB}, X)$.

Proof. Let $x = (x_1, \cdots, x_n)$ be an imputation in the core of $\Gamma(N, v^{PNE}, X)$. It therefore satisfies

$$\sum_{i=1}^{n} x_i = v(N). \tag{30}$$

$$\sum_{i \in S} x_i \geq v^{PNE}(S), \forall S \subseteq N. \tag{31}$$

By the above proposition we have $v^{PNE}(S) > v^{PB}(S)$ and therefore $\sum_{i \in S} x_i \geq v^{PB}(S), \forall S \subseteq N$. ■

This proposition shows that the core of the cooperative game $\Gamma(N, v^{PNE}, X)$ is smaller than the core of $\Gamma(N, v^{PB}, X)$. The implication is that an IEA agreement which is in the core of $\Gamma(N, v^{PB}, X)$ and not in the core $\Gamma(N, v^{PNE}, X)$ may be contested on the ground that the way that the payoffs are computed is not the right one. Actually, the smallest core is not a singleton and some bargaining must still take place to pick up the IEA that will indeed be implemented. Given that both games are convex, it is tempting to adopt the Shapley value as a solution to the cooperative game. The merits of such a solution include the facts of it being a singleton Pareto-optimal imputation, a fair one and for being at the center of gravity of the core when the game is convex. Given this and the result of the last proposition, it is obvious that the Shapley values of the cooperative games $\Gamma(N, v^{PNE}, X)$ and $\Gamma(N, v^{PB}, X)$ coincide. Recall that the Shapley value is given by

$$\phi_i(v^x) = \sum_{S \ni i, S \subseteq N} \frac{(s-1)!\,(n-s)!}{n!} \left(v^x(S) - v^x(S \setminus \{i\})\right).$$

To obtain these numbers, one needs of course to fix the number of players and compute the marginal contribution of player i to coalition S, that is $v^x(S) - v^x(S \setminus \{i\})$. The Shapley value of a player is a weighted average of his marginal contributions to all coalitions that he may join.

4. Stability of coalitions

The stable coalitions literature dealing with the design of an IEA for pollution control has shown that, irrespective of the number of players and with or without monetary transfers, only a very small number of countries will actually sign such an agreement. The reason is that a country can do better (higher payoff) by letting the others sign such an agreement and continue itself to pollute at his (higher) noncooperative level. In this approach an IEA is seen as a voluntary agreement and a country will adhere to it if it is profitable to do so. It is important here to stress that the number of players, i.e., the countries part of the IEA, is endogenous.

The cooperative game approach proceeds differently. Indeed, once the set of players, their strategies and their payoffs are defined, the problem becomes one of designing a mechanism to share the gain of cooperation between the players. A "nice" allocation can be found or not but the point is that this allocation is sought from the perspective of the grand coalition. The computation of the characteristic function values of all other coalitions is necessary to assess what would be acceptable to a coalition given its strategic force. In this methodological framework, the set of involved countries is given.

It is of interest as put forward by Tulkens (1998) to see if the two approaches can be reconciled and if not to see which characteristic function is more inclusive, i.e., leads to a larger stable coalition.

Recalling that $W_i^x(S)$ is the (maximized) payoff of player $i \in S$ and $Y_j^x(S)$ is the (maximized) payoff of $j \in N \backslash S$, we have the following definition.

Definition 17 *A coalition $S \subseteq N$ is stable if it satisfies the following conditions*

$$(i) \qquad Y_i^x(S \backslash \{i\}) - W_i^x(S) < 0, \forall i \in S \quad \textit{(exit test).} \tag{32}$$

$$(ii) \qquad W_j^x(S \cup \{j\}) - Y_j^x(S) < 0, \forall j \in N \backslash S \quad \textit{(entry test).} \tag{33}$$

Condition (i) states that there is no incentive to defect for all countries belonging to coalition S. Condition (ii) states that there is no incentive to broaden the coalition for all countries not in S.

The computation of the characteristic function values provides the outcome of coalitions and what remains to be done is to allocate this total outcome among the players belonging to these coalitions. To simplify this task, we shall assume from now on that players are, as in Carraro and Siniscalco (1992), symmetric, that is $b_i = b$ and $d_i = d$ for all players. Under symmetry, all players get obviously an equal share that is $W_i^x(S) = \frac{1}{s}W^x(S)$, $x = PNE, NE, PB$.

Proposition 18 *If the outcomes of coalitions are computed under PNE mode of play and the players are symmetric, then no stable coalition exists.*

Proof. Under the symmetry assumption, (14) and (15) become

$$\begin{aligned} W_i^{PNE}(S) &= \frac{1}{2}b^2 - ndb + \frac{d^2}{2}\left(s^2 + 2(n-s)\right), \\ Y_j^{PNE}(S) &= \frac{1}{2}\left(b^2 - d^2\right) - ndb + \left(s^2 + n - s\right)d^2. \end{aligned}$$

To check for exit test, compute:

$$Y_i^{PNE}(S\backslash\{i\}) - W_i^{PNE}(S) = \frac{d^2}{2}(s-1)(s-3),$$

which is negative for $s < 3$.

To check for entry test, compute:

$$W_j^{PNE}(S \cup \{j\}) - Y_j^{PNE}(S) = \frac{d^2 s}{2}(2-s)$$

which is negative for $s > 2$. Hence the result. ■

Proposition 19 *If the outcomes of coalitions are computed under NE mode of play, the players are symmetric and the number of players is at least 3, then the only stable coalition is formed of s players where $\frac{n-1}{2} < s < \frac{n+1}{2}$*

Proof. Under the symmetry assumption, (20) and (21) become

$$W_i^{NE}(S) = \frac{1}{2}b^2 + -ndb + \frac{1}{2}\left(s^2 + 2(n-s)^2\right)d^2,$$

$$Y_j^{NE}(S) = \frac{1}{2}b^2 - ndb + \frac{1}{2}\left(2s^2 + (n-s)^2\right)d^2.$$

To verify for exit test, compute:

$$Y_i^{NE}(S\backslash\{i\}) - W_i^{NE}(S) = -\frac{1}{2}d^2(n-3)(n+1-2s),$$

which is negative for $s < \frac{n+1}{2}$ (assuming $n > 2$).

To check for external stability, compute

$$W_j^{NE}(S \cup \{j\}) - Y_j^{NE}(S) = \frac{1}{2}d^2(n-3)(n-2s-1),$$

which is negative for $s > \frac{n-1}{2}$ (assuming $n > 3$), and hence the result. ■

Proposition 20 *If the outcomes of coalitions are computed under* PB *mode of play and the players are symmetric, then the only stable coalition is formed of 3 players.*

Proof. Under the symmetry assumption, (28) and (29) become

$$W_i^{PB}(S) = \frac{1}{2}\left(b^2 + s^2d^2\right) - ndb,$$

$$Y_j^{PB}(S) = \frac{1}{2}b^2 - ndb + s^2d^2.$$

To verify for exit test, compute

$$Y_i^{PB}(S\backslash\{i\}) - W_i^{PB}(S) = -\frac{1}{2}d^2\left(4s - s^2 - 2\right),$$

which is negative for $s < 4$.

To verify for entry test, compute

$$W_j^{PB}(S \cup \{j\}) - Y_j^{PB}(S) = \frac{1}{2}d^2\left(2s + 1 - s^2\right),$$

which is negative for $s > 2$. Hence the result. ■

The above propositions show that when we use the outcomes $W_i^{PNE}(S)$ and $W_i^{PB}(S)$, which lead to a well defined characteristic function and a nonempty core, a stable coalition simply do not exist or is formed of 3 players. This result goes in the same direction as the findings in Carraro and Siniscalco (1992) and Barrett (1990) that an international agreement will be stable for a coalition of very small size. When we use the outcomes $W_i^{NE}(S)$, the result is less pessimistic and half of the players can be part of a stable coalition. The other half is also forming a coalition. Recall however that the adoption of Nash equilibrium to compute the outcomes did not lead to a cohesive cooperative game.

5. Conclusion

The first conclusion that can be drawn from the results is that if the cooperative games are played à la PNE or à la PB, one has suitable characteristic functions and it is possible to select, using e.g. Shapley value, the same unique imputation belonging to both cores. A cooperative game played à la NE will lack the essential property of cohesiveness and therefore this approach does not carry a sufficient incentive for cooperation between coalitions.

The second conclusion is that the two approaches, classical cooperative games and stable coalitions, cannot be reconciled in the sense that adopting one or the other does not lead to the same conclusion. We believe that these two approaches rest on so different premises that finding a full convergence does not seem to be feasible. As mentioned before, the cooperative game approach assumes an exogenous set of players to be part of an international agreement. The stable coalitions approaches sees this set as endogenous. The implication of this is that concepts such as coalitions stability and free-riding are not used at all in the same manner by both approaches. In a cooperative game, an imputation in the core is stable in the sense that no coalition can block it. From this perspective, and in particular, a player will not be better off by forming a coalition alone. Therefore, the core is considered as a solution concept that deters free-riding. This is true if the player is "stuck" in the game. If he has the option of not playing it, he can do better by letting everybody else join a grand coalition and himself free-riding, i.e., polluting at the noncooperative level. This option of leaving away (or not joining) is permitted in the stable coalitions approach with the result that free-riding will occur at a large scale.

The results of this essay rest on special functional forms for revenue and cost functions. This simplification is deliberate to focus on the main points raised in the essay. It is clearly of interest to use a more general convex damage function cost and to extend the analysis to a dynamic setting.

CONCLUDING REMARKS

The three essays of this dissertation focus on two important global environmental issues. The first issue is tropical deforestation and how the use of conditional transfers from the North to the South can help reduce the forest destruction. The second is related to the problem of free-riding and coalition stability in the context of IEA.

In the first essay we used a model representing an economic/ecological interaction of forests to show that it is possible for the North to design a subsidy program in order to help the South in managing the forest. The setting is of a differential game with a leader-follower information, played over a finite horizon. The results indicate that making the transfer function dependent on the deforestation rate directly in addition to be dependent on the forest stock, has a clear impact on slowing deforestation. This result is consistent with the findings in Van Soest and Lensink (2000). On the other hand, introducing a budget constraint faced by the North for the first time in this literature, we showed that in some cases (when the budget constraint is binding) additional conditions may be needed to ensure an improvement of the forest conservation under the subsidy program. In the case of a non effective budget constraint (when the budget in not binding), however, we also join the results of previous literature (see for example; Stahler (1996), Mohr (1996), Van soest and Lensink (2000), Martín-Herrán et al. (2002)) stating that funding forest preservation leads to its unambiguous improve. In addition, we found that the forestry countries' welfare is not always better off under the subsidy program compared to the optimal control or *laisser-faire* scenario. This means that some conditions have to be taken into consideration to guarantee the participation of the North and the South in this program. This means that signing a North-South agreement to conserve the forest is not as trivial as stated in the previous literature, which can explain the lack of concretization of such programs or agreements in real life.

The second essay's aim is to design an incentive mechanism to enforce sustainable forest management in the forestry countries. For this purpose we used an extension of the first essay's model, which shows in more details the trade-off between the agriculture and the forestry activities and introduces a natural regeneration rate of the forests. The use of incentive mechanisms is completely new to the literature on sustainable forest management. The results show that the North can encourage the forestry countries to participate in a program aiming at a better forest conservation while compensating for the revenue loss. More precisely we showed that using transfers as incentive, the donor community or the North can enforce sustainable forest exploitation in the South even if the latter is maximizing its welfare in the short-run.

Some interesting continuation in this field will be to consider efforts to regenerate the forest. This would require a model incorporating a lag structure to handle correctly the link between replanting trees and later production of the resource. The North could also contribute financially to such a program, and a Principal-agent setting would be of interest to study the effect of uncertainty on the participation constraints of both the North and the South in such a program. Further we can introduce the cost of the budget spent by the North in its objective function. Under this hypothesis the North would look for an optimal trade-off between the final forest size and the budget spent for the subsidy program.

Finally, the last essay represents a contribution to the theoretical discussion about the possibility of reconciliation between the cooperative and non-cooperative understanding of coalition's stability using characteristic function. The first conclusion that could be drawn from the results was that if the cooperative games are played à la PNE or à la PB, one has suitable characteristic functions and it is possible to select, using e.g. Shapley value, the same unique imputation belonging to both cores. A cooperative game played à la NE will lack the essential property of cohesiveness and therefore this approach does not carry a sufficient incentive for cooperation between coalitions.

The second and main conclusion was that the two approaches, classical cooperative games and non-cooperative games, cannot be reconciled in the sense that adopting one or the other does not lead to the same conclusion. We believe that these two approaches rest on so different premises that finding a full convergence does not seem to be feasible. In fact, it is difficult to deter free riding and there is no large coalition that can emerge, if countries play non-cooperatively. This result is again proved in reality if we confront it with the unfolding events we have seen through the development of the Kyoto coalition.

However, the results of this essay rest on special functional forms for revenue and cost functions. This simplification is deliberate to focus on the main points raised in this essay. It is clearly of interest to use a more general convex damage function cost and to extend the analysis to a dynamic setting.

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